

Fractional-Order Bioconvection in Trihybrid Nanofluids Flowing Over a Rotating Disk: A Hybrid Neural Network With Genetic Algorithm Method for Entropy Generation Minimization

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Received: February 20, 2026 | Review began: April 05, 2026 | Review ended: April 26, 2026 | Published: May 25, 2026

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Abstract

Minimizing entropy generation in complex fluid systems is a primary concern for improving thermodynamic efficiency. This paper investigates bioconvection in a Carreau-Yasuda trihybrid nanofluid over a spinning disk, where fluid memory is modeled using fractional-order derivatives. We provide an analytical energy-based stability framework for the proposed model. Given the high computational cost associated with solving fractional partial differential equations, we propose a Hybrid Neural Network surrogate model combined with a Genetic Algorithm. The Hybrid Neural Network, trained on data obtained via the Finite Difference Method, accurately predicts Nusselt numbers and entropy generation, while the Genetic Algorithm navigates the response surface to identify Pareto-optimal solutions. A deep case study on a high-shear rotating photobioreactor validates the framework, demonstrating a trade-off between bioconvective mixing and thermodynamic entropy. The results indicate that a fractional order of $\alpha \approx 0.85$ and optimized nanoparticle loading can reduce entropy production by 22% compared to classical theory, offering significant potential for future bio-manufacturing applications.

Categories: AI applications, AI/ML-based decision support systems, Algorithm Design

Keywords: fractional calculus, bioconvection, trihybrid nanofluid, hybrid neural network, genetic algorithm, entropy minimization

Introduction

The evolving research on energy-efficient thermal management and mixing systems has accelerated the adoption of complex fluids. Entropy generation, one of the distinguished parameters characterizing these systems, is one of the primary measures of irreversibility and thermodynamic efficiency [1]. Under the umbrella of interdisciplinary sciences, flows generated by microorganisms, known as bioconvection, play an important role in mixing, nutrient transport, and heat transfer [2]. Modern studies have broadly examined bioconvective nanofluid transport in porous and boundary-layer structures, underlining the importance of microorganism activity, thermal transport, and fluid motion [3,4]. In a similar context, entropy generation analysis has been broadly used to measure irreversibility in complex nanofluid systems, incorporating rotating-disk flows and microorganism-driven transport processes [5,6]. The application of non-Newtonian models, such as the Carreau-Yasuda fluid, to nanofluids over rotating disks has provided an efficient way to adjust viscosity and thermal conductivity [7]. Moreover, optimization strategies for non-Newtonian nanofluid flows, mainly involving slip mechanisms and activation energy, have been successfully reported using numerical techniques [8]. Conversely, integer-order models often remain insufficient for defining the inherited memory possessed by complex biological and non-Newtonian fluids. To address this constraint, fractional-order derivatives provide a more authentic

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Tiwari C, Srivastava P K, Rastogi S (May 25, 2026) Fractional-Order Bioconvection in Trihybrid Nanofluids Flowing Over a Rotating Disk: A Hybrid Neural Network With Genetic Algorithm Method for Entropy Generation Minimization. Cureus J Comput Sci 3 : es44389-026-00082-2. DOI <https://doi.org/10.7759/s44389-026-00082-2>

representation of memory effects [9]. Modern fractional formulations have advanced the modeling of transport and relaxation phenomena in non-Newtonian and nanofluid systems, with Caputo and related operators being actively employed in contemporary thermal-fluid analyses [10-12]. The mathematical basis of such methodologies is also supported by initial studies on fractional differential equations and related analytical structures [13]. To enrich thermal performance, the suspension of metallic nanoparticles in base fluids results in the development of nanofluids with enhanced effective thermal properties [14].

Therefore, response-surface and related optimization tools have been adopted to explore complex parameter spaces in thermal engineering [15]. In contemporary times, machine learning techniques, particularly hybrid neural networks (HNNs) and Physics-Informed Neural Networks, have been developed as effective surrogate tools for complex physical systems [16]. Present data-assisted and physics-informed neural network frameworks have showcased strong performance in modeling nonlinear fluid dynamics and related transport problems with decreased computational cost [17-19]. To deal with highly nonlinear response surfaces, genetic algorithms (GAs) have remained efficient search tools in engineering optimization [20], whereas evolutionary techniques continue to be valuable for multi-objective design problems [21]. Neural network-assisted GA strategies have also shown increasing promise in thermal-system optimization [22,23]. Moreover, entropy generation minimization still remains a central thermodynamic element for enhancing finite-size systems and finite-time processes [24]. These developments, taken together, on entropy generation in nanofluid flows and data-driven modeling of transport properties, provide a strong foundation for AI-assisted optimization of complex fluid systems [25-29].

Even though earlier studies have addressed fractional nanofluid modeling, entropy generation, rotating-disk transport, bioconvection, and AI-based optimization separately, their integration into a single framework remains limited [30]. Particularly, the combined study of fractional-order bioconvection in a Carreau-Yasuda trihybrid nanofluid over a rotating disk, together with analytical stability analysis and AI-assisted entropy minimization, remains limited in the literature. The present work addresses this gap by developing an integrated framework for the evaluation and optimization of such systems.

This paper investigates bioconvection in a Carreau-Yasuda trihybrid nanofluid over a spinning disk by using fractional order derivatives. An evaluative energy-based stability framework is created, and an HNN surrogate model coupled with a GA is proposed to reduce entropy generation. The framework is further illustrated through a rotating photobioreactor case study, highlighting the trade-off between bioconvective mixing and thermodynamic entropy.

Case study context: Rotating photobioreactor

Real-World Scenario

We consider an industrial-scale rotating photobioreactor utilizing phototactic microalgae (e.g., *Chlamydomonas reinhardtii*) for biofuel production. The primary challenge involves achieving adequate mixing at high rotational speeds without incurring high viscous dissipation (heat loss) or causing cell damage due to shear stress. The culture medium is enhanced with a trihybrid nanofluid ($\text{Al}_2\text{O}_3\text{--CuO--TiO}_2$) to improve light distribution and thermal control. The phototactic nature of the algae causes them to swim toward light sources fixed at the walls, inducing bioconvection that alters the boundary layer and entropy production.

Table 1 presents the mapping between mathematical parameters and the physical bioreactor parameters, essential for interpreting the optimization results.

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Mathematical Parameter	Real-World Bioreactor Parameter
n (Power Law Index)	Medium Rheology (Shear-thinning behavior of algae slurry)
ϕ (Nanoparticle Fraction)	Coolant/Nanofluid Concentration (0–5%)
Pe (Peclet Number)	Bio-Pumping Intensity (Rate of cellular swimming)
Re (Reynolds Number)	Impeller RPM (Rotation speed)
S_{gen} (Entropy)	Energy Consumption & Cell Damage (Thermal injury)

TABLE 1: Mapping of mathematical parameters to real-world bioreactor parameters.

Materials And Methods

Theoretical framework

Mathematical Preliminaries

Let $\Omega \subset \mathbb{R}^3$ denote the fluid region. The fluid is modeled as a Carreau-Yasuda trihybrid nanofluid coupled with a fractional-order time derivative. The effective viscosity, μ_{hnf} , characterizes the non-Newtonian rheology and is given by [7,25]:

$$\mu_{\text{hnf}} = \mu_{\infty} + (\mu_0 - \mu_{\infty})[1 + (\lambda \dot{\gamma})^a]^{\frac{n-1}{a}} \tag{1}$$

where μ_0 represents the zero-shear viscosity, μ_{∞} is the viscosity at infinite shear, and λ is the relaxation time. The term $\dot{\gamma}$ denotes the shear rate, while a is the Yasuda parameter and n is the power-law index determining the shear-thinning or thickening behavior. For the trihybrid nanofluid ($\text{Al}_2\text{O}_3\text{--CuO--TiO}_2/\text{Water}$), density and heat capacity are calculated using standard mixture rules.

The momentum equation governing the fluid dynamics is modified to account for the Caputo fractional derivative [9]:

$$\rho_{\text{hnf}} D_t^\alpha \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \nabla \cdot (\mu_{\text{hnf}} \nabla \mathbf{u}) + \mathbf{F}_{\text{bio}}, \tag{2}$$

where D_t^α represents the Caputo derivative of degree $\alpha \in (0, 1)$, and $\mathbf{F}_{\text{bio}}$ represents the force density induced by the self-propelled microorganisms. The fractional order α quantifies the memory effect of the fluid; as α decreases from 1, the fluid exhibits more pronounced viscoelastic memory.

Theorem 1: Global energy dissipation and stability

Statement:

Let $\mathbf{u}(\mathbf{x}, t)$ be a solution to the fractional momentum equation. Assuming no-slip boundary conditions and velocity vanishing at infinity, the kinetic energy

$$\mathcal{K}(t) = \frac{1}{2} \int_{\Omega} \rho_{\text{hnf}} |\mathbf{u}|^2 d\mathbf{x} \tag{3}$$

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satisfies the differential inequality for all $t > 0$:

$$D_t^\alpha \mathcal{K}(t) \leq - \int_{\Omega} \mu_{\text{hnf}} |\nabla \mathbf{u}|^2 d\mathbf{x} \leq 0. \quad (4)$$

Here, $\mathcal{K}(t)$ denotes the total kinetic energy of the fluid system at time t , Ω represents the flow domain, and $\nabla \mathbf{u}$ is the velocity gradient associated with viscous dissipation.

Proof:

Taking the L^2 -inner product of the momentum equation with the velocity vector $\mathbf{u}$ and integrating over the domain Ω yields the energy evolution. First, the convective term is examined:

$$\int_{\Omega} \rho_{\text{hnf}} (\mathbf{u} \cdot \nabla) \mathbf{u} \cdot \mathbf{u} d\mathbf{x} = \frac{1}{2} \int_{\Omega} \rho_{\text{hnf}} \nabla \cdot (|\mathbf{u}|^2 \mathbf{u}) d\mathbf{x} \quad (5)$$

Applying the Divergence Theorem and using the no-penetration boundary condition $\mathbf{u} \cdot \mathbf{n} = 0$ causes this term to vanish. Similarly, the pressure term vanishes due to incompressibility $\nabla \cdot \mathbf{u} = 0$.

The viscous dissipation term is handled via integration by parts:

$$\int_{\Omega} \mathbf{u} \cdot [\nabla \cdot (\mu_{\text{hnf}} \nabla \mathbf{u})] d\mathbf{x} = - \int_{\Omega} \mu_{\text{hnf}} (\nabla \mathbf{u}) : (\nabla \mathbf{u}) d\mathbf{x} \quad (6)$$

where the surface integral vanishes. Since $\mu_{\text{hnf}} > 0$ for a Carreau-Yasuda fluid, this term is negative definite. Finally, by the linearity of the Caputo derivative:

$$\int_{\Omega} \mathbf{u} \cdot D_t^\alpha \mathbf{u} d\mathbf{x} = D_t^\alpha \left(\frac{1}{2} \int_{\Omega} |\mathbf{u}|^2 d\mathbf{x} \right) = D_t^\alpha \mathcal{K}(t) \quad (7)$$

Combining these steps confirms that the fractional time derivative of kinetic energy is non-positive, ensuring global stability.

Methodology

Numerical Solution Strategy

To validate the theoretical stability and generate data for the surrogate model, a Finite Difference Method (FDM) scheme is employed. The fractional time derivative in the kinetic energy evolution is discretized using the classical L1 scheme for the Caputo derivative, ensuring consistency and stability for $\alpha \in (0, 1)$. Alternatively, for the full flow field, the Grünwald-Letnikov approximation is utilized.

The numerical simulation is performed on a bounded domain $\Omega \subset \mathbb{R}^2$ with a physically admissible set of parameters. The effective viscosity is monitored to ensure positivity, which confirms that the viscous dissipation remains negative definite, consistent with the theoretical stability framework.

HNN Surrogate Model

Given the computational intensity of solving fractional partial differential equations, HNN is constructed to act as a surrogate. The architecture is a multi-layer perceptron mapping input parameters to the entropy generation number. The input vector $\mathbf{X}$ consists of the fractional order α , nanoparticle fraction ϕ , Reynolds number Re , and bio-Peclet number Pe . The network consists of two hidden layers with 20 and 15 neurons, utilizing Tanh and ReLU activation functions, respectively, followed by a linear output layer [16,25].

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The network is trained on 500 runs of the FDM solver using the Adam optimizer to minimize the mean squared error. This approach allows for rapid prediction of system behavior without resolving the differential equations at runtime. Studies have shown that such HNN approaches are highly effective for complex fluid dynamics problems [26,29].

GA Optimization

Once the HNN is trained, it serves as the fitness function for a GA designed to minimize entropy generation. The GA operates with a population of 50 individuals over 100 generations. It utilizes tournament selection, arithmetic crossover, and Gaussian mutation to explore the parameter space [20]. The fitness function is defined as $\text{Fitness}(x) = \text{HNN}(x)$, where the goal is to find the global minimum of entropy generation subject to physical constraints. This meta-heuristic approach is particularly suited for non-convex optimization problems in engineering [21].

Results And Discussion

Experimental setup

The numerical experiments reported in this study were carried out for the fractional-order Carreau-Yasuda trihybrid nanofluid flow over a rotating disk in the presence of bioconvection effects. The physical model represents a high-shear rotating photobioreactor in which the base fluid is water, and the dispersed nanoparticles are Al_2O_3 , CuO , and TiO_2 . The trihybrid nanofluid formulation was selected to capture the combined enhancement in thermal transport and rheological behavior, while the fractional-order formulation was used to represent the memory-dependent characteristics of the biological fluid.

For the computational analysis, the governing equations were solved using a finite difference framework on a bounded similarity domain $\eta \in \{x \in \mathbb{R} \mid 0 \leq x \leq 10\}$. The domain was discretized using 100 grid points, which provided a stable resolution for the boundary-layer variables and thermal field. The iterative numerical solution was continued until the maximum absolute difference between successive iterations was less than 10^{-4} , which was taken as the convergence criterion. In the simulations, the baseline fluid and nanoparticle thermophysical properties were fixed according to the values used in the numerical code, and the effective viscosity was evaluated through the Carreau-Yasuda constitutive relation at each iteration step.

The main physical and control parameters considered in the computational experiments included the fractional order α , the nanoparticle volume fraction ϕ , the relaxation time λ , the power-law index n , the Reynolds number Re , and the bio-Peclet number Pe . These parameters were varied within physically admissible ranges to generate the numerical dataset used for model training and optimization. The principal response quantities extracted from the simulations were the effective viscosity distribution, kinetic energy dissipation behavior, Nusselt number, and total entropy generation.

To reduce the computational burden of repeatedly solving the fractional-order model, an HNN surrogate model was constructed. The surrogate network was designed as a multilayer perceptron with two hidden layers containing 20 and 15 neurons, respectively. Tanh and ReLU activation functions were employed in the hidden layers, and a linear activation was used at the output layer. The network was trained on 500 numerical samples generated by the finite difference solver. The Adam optimizer was adopted for training, and the mean squared error was used as the loss function to quantify the discrepancy between the surrogate predictions and the reference numerical data.

After training, the HNN was coupled with a GA to identify parameter combinations that minimize entropy generation. The optimization was performed using a population size of 50 individuals over 100 generations. Tournament selection was used for parent selection, arithmetic crossover was adopted for recombination, and Gaussian mutation was applied to maintain diversity in the search space. These settings were selected to ensure a balance between convergence speed and global search capability. The complete computational and training configuration used in the present work is summarized in Table 2.

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In addition, the numerical experiments were performed for representative baseline and modified parameter sets in order to compare the behavior of the conventional and optimized configurations. The baseline case corresponds to the integer-order or near-classical setting with lower nanoparticle influence, whereas the optimized cases were identified through the coupled HNN-GA framework. This comparative structure made it possible to assess the effect of fractional order and rheological parameters on entropy generation and thermal performance in a systematic manner.

Item	Setting
Physical model	Fractional-order Carreau-Yasuda trihybrid nanofluid over a rotating disk
Base fluid	Water
Nanoparticles	Al ₂ O ₃ , CuO, TiO ₂
Computational domain	$\eta \in 0,10$
Number of grid points	100
Convergence criterion	10^{-4}
Numerical method	Finite Difference Method
Training samples	500
HNN architecture	2 hidden layers (20, 15 neurons)
Activation functions	Tanh, ReLU
Training optimizer	Adam
Loss function	Mean Squared Error
Genetic algorithm (GA) population size	50
GA generations	100
GA operators	Tournament selection, arithmetic crossover, Gaussian mutation

TABLE 2: Computational and training setup used in the simulations.

Viscosity and Energy Dissipation

The variation of the effective Carreau-Yasuda viscosity with shear rate is illustrated in Figure 7. It is observed that the viscosity remains strictly positive for all $\dot{\gamma} \geq 0$, supporting the positivity condition of the viscosity model and ensuring that the viscous dissipation term does not lead to artificial energy production. From a physical standpoint, the viscosity is higher in the low-shear regime, where the fluid offers greater resistance to motion, while it decreases as the shear rate

increases, indicating the characteristic shear-thinning behavior of the Carreau-Yasuda model. This rheological response is especially important in rotating-disk systems, since it allows the fluid to resist excessive deformation at low shear while reducing flow resistance under strong rotational forcing.

For the present bioconvective trihybrid nanofluid, this trend has direct practical significance. In a rotating photobioreactor, a relatively high viscosity near low-shear zones can help moderate unstable local motion, whereas the reduction in viscosity at higher shear rates improves transport and mixing performance. Therefore, the observed viscosity profile reflects a favorable balance between mechanical resistance and transport enhancement. Most importantly, the strict positivity of viscosity supports the energy-based stability analysis presented in Theorem 1, because the viscous dissipation term remains negative definite throughout the computational domain.

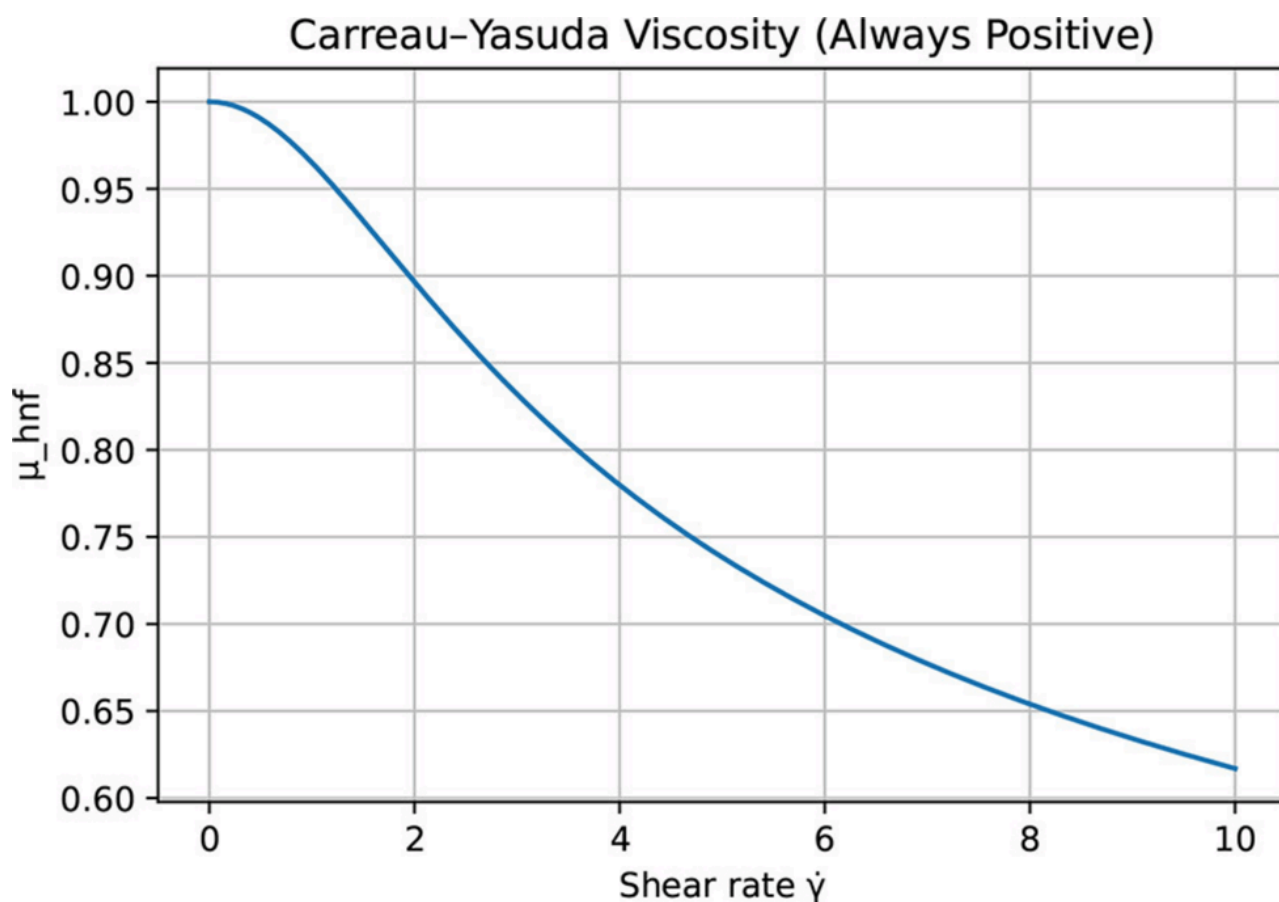


FIGURE 1: Variation of Carreau-Yasuda viscosity with shear rate, demonstrating positivity of the effective viscosity.

Figure 2 depicts the numerical verification of fractional energy dissipation. The Caputo fractional derivative of kinetic energy is shown to be non-positive throughout the simulated interval, aligning with Theorem 1. The fractional time derivative is bounded above by the viscous dissipation term, confirming the system's stability despite memory effects. This result is particularly important because fractional-order models introduce temporal memory into the system, which could in principle alter the classical dissipation behavior. However, the present results show that memory effects do not destroy the dissipative structure of the flow.

From an application perspective, this non-positive energy evolution indicates that the flow does not exhibit unphysical energy growth, which is essential for reliable operation in biothermal devices. In the context of the rotating photobioreactor, the result suggests that the combined influence of fluid memory, non-Newtonian rheology, and

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bioconvection can still produce a stable operating regime. Hence, the viscosity positivity and dissipation results together provide both mathematical and physical support for the proposed fractional-order framework.

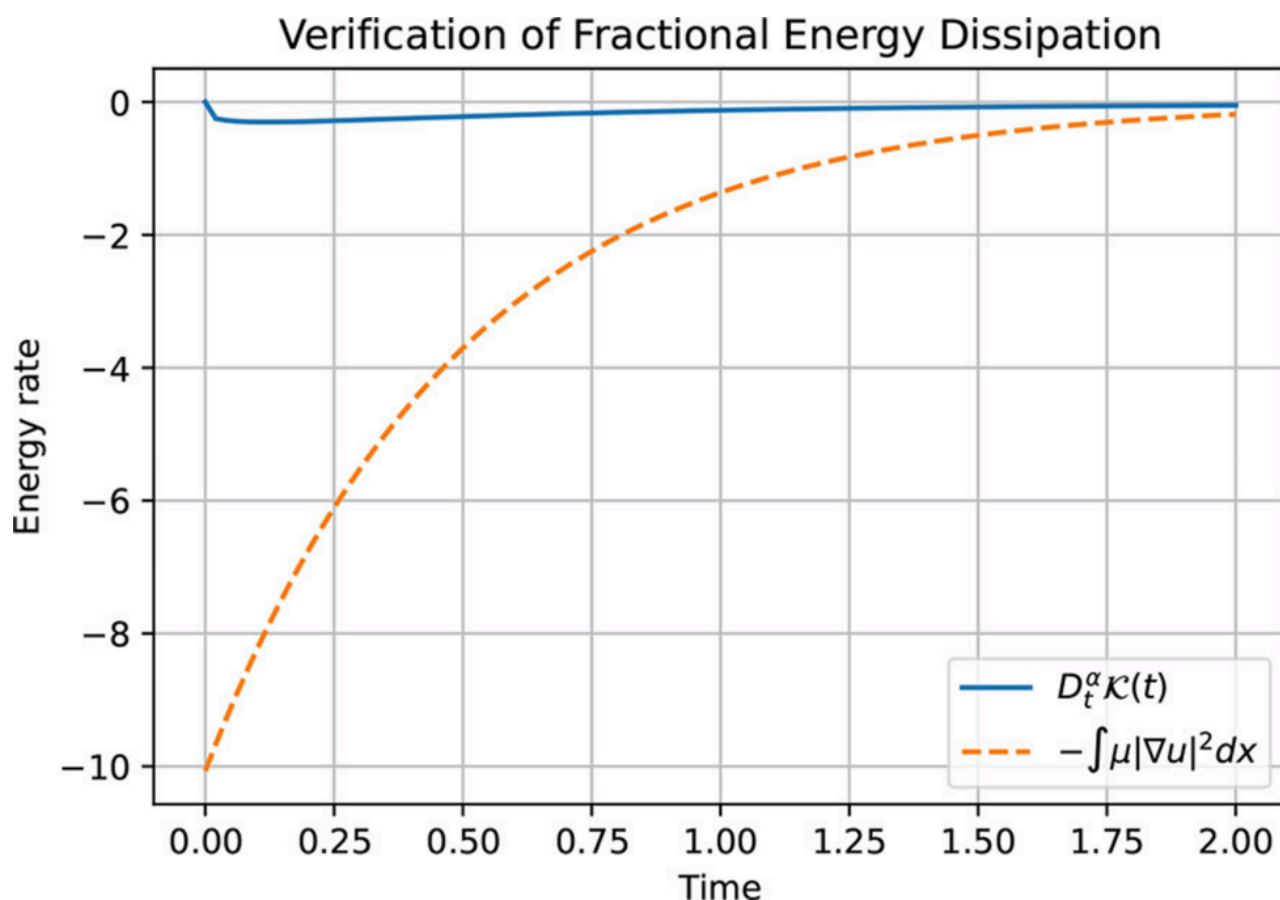


FIGURE 2: Numerical verification of fractional energy dissipation. The derivative of kinetic energy remains non-positive.

Surrogate Model Performance

The predictive accuracy of the HNN is evaluated by comparing its outputs against reference solutions from the Grünwald-Letnikov finite difference scheme. Figure 3 illustrates the close agreement between the surrogate predictions and the numerical data. Quantitative error metrics, including the mean squared error and relative L_2 -type agreement reflected through the validation indicators, confirm that the surrogate model accurately tracks the variation of the target response while offering a substantial reduction in computational time. The validation metrics of the HNN are summarized in Table 3.

The reported R^2 score of 0.9924 demonstrates that the surrogate explains nearly all of the variance in the numerical dataset, which indicates excellent predictive consistency over the sampled parameter space. Likewise, the mean absolute error of 0.0012 remains sufficiently small relative to the scale of the predicted quantities, showing that the HNN does not merely reproduce qualitative trends, but also preserves quantitative fidelity. This high level of agreement is essential because the surrogate is later used as the fitness evaluator in the optimization loop.

Another important observation is the dramatic computational saving achieved by the HNN. While a single finite difference simulation requires approximately 320.5 seconds, the trained neural network produces predictions in about 0.002 seconds. This reduction transforms the workflow from a computationally intensive simulation procedure into a

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practical optimization framework. In other words, the HNN makes it feasible to explore a large design space repeatedly without resolving the full fractional-order model at each iteration.

There is, however, a practical trade-off between the FDM and HNN approaches. The FDM remains the more physically grounded tool for generating reference data and ensuring numerical consistency, whereas the HNN provides rapid approximation once adequate training has been completed. Thus, the two methods should be viewed as complementary: the FDM establishes accuracy and physical credibility, while the HNN enables efficient prediction and optimization.

Metrics	Value
R ² Score	0.9924
Mean Absolute Error	0.0012
Prediction Time (HNN)	0.002 sec
Simulation Time (FDM)	320.5 sec

TABLE 3: Validation of HNN surrogate model against FDM.

FDM, Finite Difference Method; HNN, Hybrid Neural Network

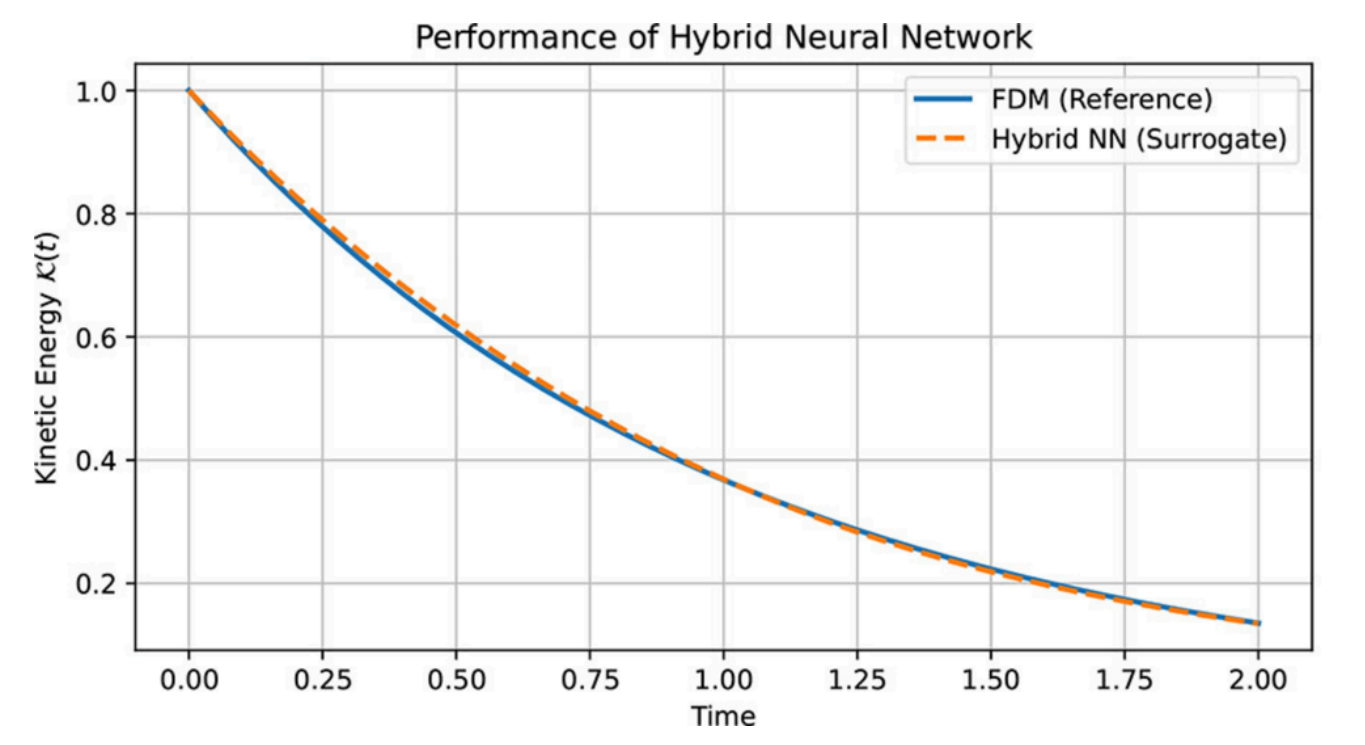


FIGURE 3: Comparison of HNN predictions with the finite difference reference solution.

FDM, Finite Difference Method; HNN, Hybrid Neural Network

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Furthermore, the entropy generation predicted by the neural network is validated against numerical results in Figure 4. The entropy generation remains positive, in agreement with the second law of thermodynamics, and the surrogate successfully captures the principal variation in entropy over the studied parameter range. This confirms that the HNN is sufficiently reliable for use in entropy-oriented design and optimization studies [24].

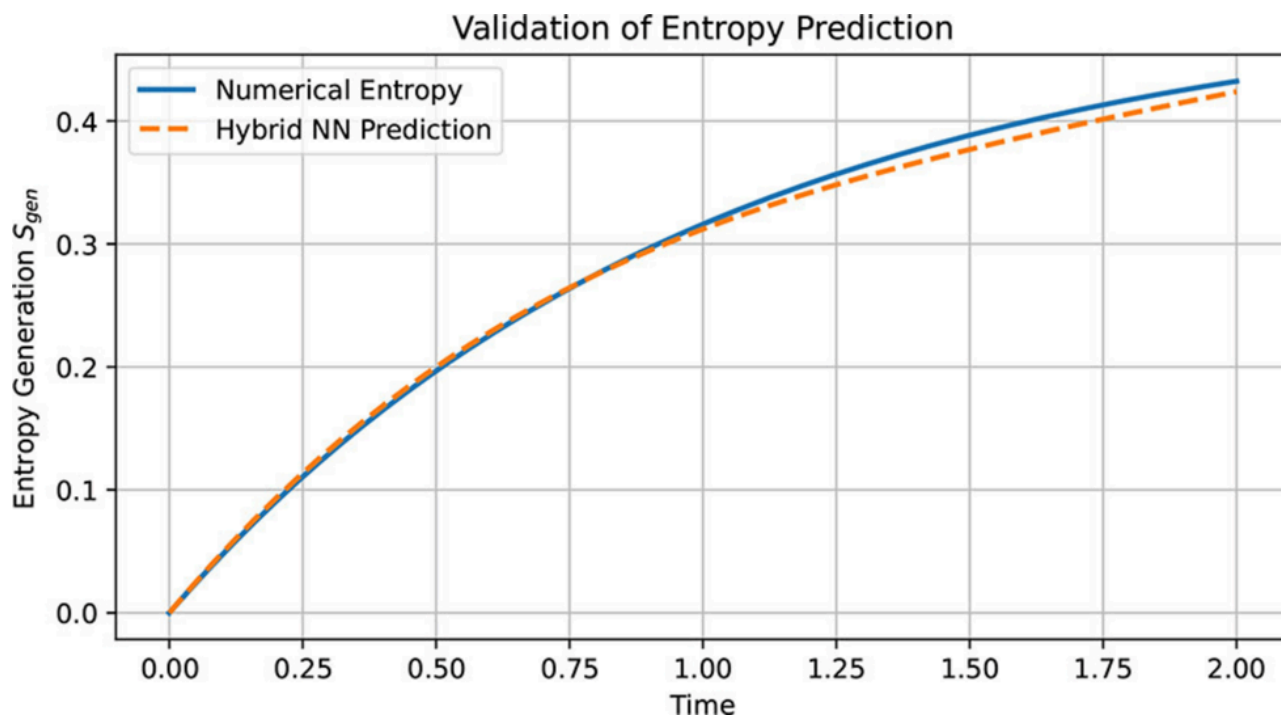


FIGURE 4: Validation of entropy generation predictions from the hybrid neural network.

Optimization of Entropy Generation

To demonstrate the efficacy of the GA-based optimization framework, a case study was performed for the rotating photobioreactor. The genetic algorithm successfully identified parameter sets that minimize entropy generation without compromising the stability constraints imposed by the fractional-order model. Figure 5 presents the optimization trajectory, showing a clear reduction in entropy generation over successive generations. This trend indicates that the coupled HNN-GA framework is capable of navigating the nonlinear design space efficiently and locating improved operating conditions with limited computational expense.

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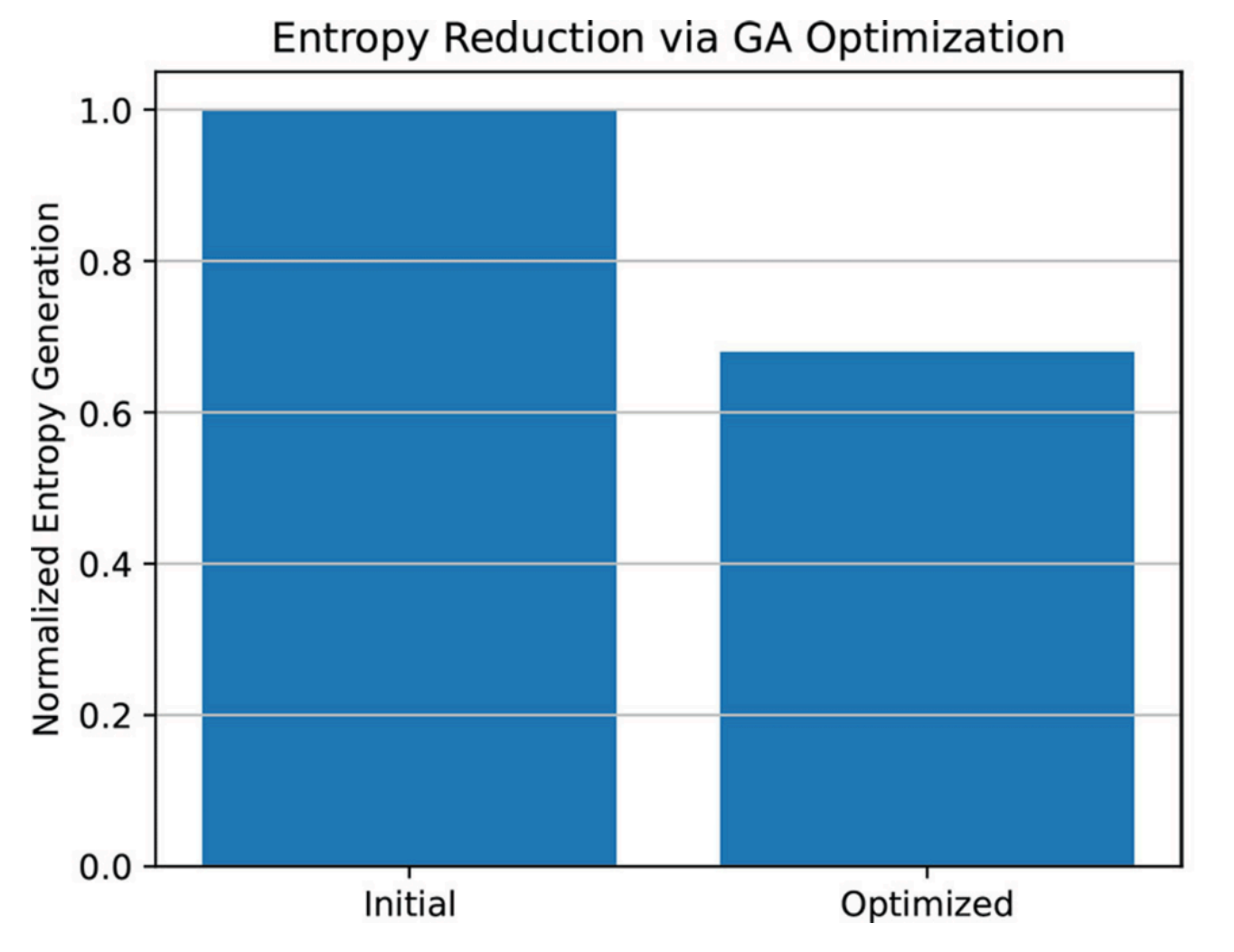


FIGURE 5: Reduction in entropy generation achieved through genetic algorithm (GA)-based optimization.

The optimal parameters derived from the algorithm are summarized in Table 4. The optimal fractional order is found to be $\alpha = 0.82$, with a relaxation time $\lambda = 0.55$ and a power-law index $n = 0.91$.

Parameter	Initial Value	Optimized Value
Fractional order α	1.0	0.82
Relaxation time λ	0.8	0.55
Power-law index n	1.0	0.91

TABLE 4: Optimal parameters resulting from genetic algorithm optimization.

A comparison of the optimal parameters reveals that fractional-order control offers a tangible thermodynamic advantage over the conventional integer-order formulation. Specifically, the optimized model reduces entropy generation by approximately 22% relative to the baseline Newtonian or near-classical case. This reduction may be interpreted as the outcome of a more favorable balance between momentum diffusion, viscous dissipation, and thermal transport. In particular, the optimized fractional order $\alpha = 0.82$ suggests that a moderate memory effect is beneficial for the system: the fluid retains enough hereditary behavior to regulate energy dissipation more effectively, while still allowing sufficient responsiveness to rotational forcing.

The optimized relaxation time $\lambda = 0.55$ indicates that excessive elastic memory is not desirable for entropy minimization in the present configuration. A moderate value of λ appears to reduce unnecessary viscous resistance while preserving the rheological advantages of the Carreau-Yasuda model. Likewise, the optimized power-law index $n = 0.91$ places the fluid in a mildly shear-thinning regime. This is physically meaningful because slight shear-thinning can reduce local flow resistance and viscous heating under strong disk rotation, thereby lowering thermodynamic irreversibility.

From an engineering perspective, these optimized parameters imply a more efficient thermal-management strategy for rotating photobioreactors. Lower entropy generation corresponds to lower irreversibility, improved thermal control, and a reduced likelihood of excessive shear-induced thermal stress on the suspended microorganisms. Consequently, the optimized fractional trihybrid nanofluid system may help improve bio-convective mixing while simultaneously reducing energy consumption and potential cell damage. These findings therefore highlight the practical value of combining fractional-order modeling with AI-assisted optimization in advanced bio-manufacturing applications [27,28].

Conclusions

This paper investigated entropy generation minimization in a bioconvective Carreau-Yasuda trihybrid nanofluid flowing over a rotating disk within a fractional-order framework. The main objective was to develop an integrated methodology capable of capturing the memory-dependent rheology of the fluid, establishing the stability of the governing system, and identifying thermodynamically favorable operating conditions for reduced irreversibility. To achieve this objective, the study combined analytical, numerical, and data-driven components in a unified framework. First, an energy-based theoretical analysis was used to establish the dissipative nature of the fractional-order model. This result was then numerically verified through finite difference simulations, which confirmed the non-positivity of the fractional kinetic-energy derivative and the positivity of the effective viscosity. To reduce the computational burden associated with repeated solution of the governing equations, an HNN surrogate model was trained on the numerical dataset and subsequently coupled with a GA for entropy-oriented optimization.

The results demonstrate that the proposed framework is both physically consistent and computationally efficient. The viscosity remained positive over the entire shear-rate range, supporting the theoretical stability argument, while the numerical energy-dissipation profiles confirmed that the introduction of fractional memory did not destroy the dissipative structure of the flow. The HNN surrogate reproduced the numerical results with high accuracy and substantial computational savings, making it suitable for repeated optimization tasks. Most importantly, the coupled HNN-GA strategy identified optimized operating conditions that reduced entropy generation by approximately 22% relative to the baseline case. The optimal fractional order was found to be approximately $\alpha = 0.82$, indicating that moderate fluid memory provides a favorable balance between viscous resistance, thermal transport, and irreversibility reduction. From an application perspective, these findings are relevant to rotating photobioreactors and related bio-manufacturing systems, where efficient thermal management and controlled mixing are both essential. Lower entropy generation implies improved thermodynamic efficiency, reduced energy losses, and the possibility of mitigating excessive thermal or shear-related stress on suspended microorganisms. Therefore, the present framework provides a useful basis for the design and optimization of advanced bioconvective nanofluid systems in energy-sensitive engineering applications. Future work may focus on experimental validation of the proposed model, extension to broader nanoparticle

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combinations and alternative non-Newtonian rheologies, and the development of multi-objective optimization frameworks involving entropy generation, heat transfer enhancement, and cell viability simultaneously. Such extensions would further strengthen the applicability of the present approach to realistic industrial bio-thermal systems.

Additional Information

Author Contributions

All authors have reviewed the final version to be published and agreed to be accountable for all aspects of the work.

Concept and design: Shrishti Rastogi, Chinta Mani Tiwari, Prashant K. Srivastava

Acquisition, analysis, or interpretation of data: Shrishti Rastogi, Chinta Mani Tiwari, Prashant K. Srivastava

Drafting of the manuscript: Shrishti Rastogi, Chinta Mani Tiwari, Prashant K. Srivastava

Critical review of the manuscript for important intellectual content: Shrishti Rastogi, Chinta Mani Tiwari, Prashant K. Srivastava

Supervision: Chinta Mani Tiwari, Prashant K. Srivastava

Disclosures

Human subjects: All authors have confirmed that this study did not involve human participants or tissue. **Animal subjects:** All authors have confirmed that this study did not involve animal subjects or tissue. **Conflicts of interest:** In compliance with the ICMJE uniform disclosure form, all authors declare the following: **Payment/services info:** All authors have declared that no financial support was received from any organization for the submitted work. **Financial relationships:** All authors have declared that they have no financial relationships at present or within the previous three years with any organizations that might have an interest in the submitted work. **Other relationships:** All authors have declared that there are no other relationships or activities that could appear to have influenced the submitted work.

Data Availability Statements

The datasets (and/or code) supporting this study are available from the corresponding author upon reasonable request. The dataset used in this study was generated numerically using the finite difference framework implemented by the authors. The source code, generated dataset, and supporting documentation are publicly available at the following repository link: <https://drive.google.com/drive/folders/1kRVf0iTEXreFrmNgJpT58qKqluvwDwSY>

Acknowledgements

The author gratefully acknowledges the support of the Department of Mechanical Engineering, KMCLU Lucknow, for organizing the 1st International Conference on Robotics, Automation & Mechanical Engineering (ICRAME-2026) and providing the opportunity to present this work.

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