

Multistage Compression Technique for Electrocardiogram Signals with R-Peak Detection for Internet of Things Healthcare

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Abstract

Telemedicine and remote patient monitoring (RPM) are rapidly evolving fields transforming how healthcare is delivered. Remote monitoring solutions enable more efficient and effective care by connecting patients with healthcare providers in real-time with Internet-of-Things (IoT) enabled devices. However, the large scale of data transmission creates congestion in network traffic. This needs to be taken care of by applying a suitable data compression scheme for more effective and timely communication. In this work, the authors considered a two-level compression method for secure and efficient data transmission. Initially, the Wavelet Transform method utilizing the Daubechies mother wavelet is applied for noise suppression, data compression, and efficient encoding. Additionally, a novel algorithm is developed to convert 1-dimensional data to 2-dimensional data for enhanced analysis. Subsequently, the Singular Value Decomposition technique is employed to further reduce data size while maintaining compression efficiency. The proposed methodology demonstrates excellent performance for IoT-based communication, achieving a compression ratio of 98.2, percentage root mean square difference of 1.2034, quality score of 82.345, and signal-to-noise of 4.1858, and peak signal-to-noise ratio of 1.8013. These results highlight the effectiveness for telemedicine applications.

Categories: Signal Processing and Image Analysis, Embedded Systems and IoT, Wireless Communication Systems

Keywords: telemedicine, ecg-compression, wavelet-transform, singular value decomposition, encoding, cr, prd, snr, psnr

Introduction

Internet of Things (IoT) based system [1] can serve efficiently by reducing the burden on hospitals due to population growth. When doctors provide medications and a designated dosage plan, they can oversee if patients follow the plan by utilizing linked devices. IoT-based monitoring is an alternate solution to this problem. One significant aspect of incorporating IoT into connected devices is that doctors can verify adherence. In this IoT monitoring, the Electrocardiogram (ECG) signal [2] plays an important role in diagnosing different heart conditions, monitoring patient health, and guiding medical interventions. There comes the efficient storage, transmission, and analysis of cardiac data. A need has arisen for better compression due to the rise of wearable devices, telemedicine platforms, and remote patient monitoring systems that can handle more ECG data that is being generated. The significance of ECG signal compression lies in its ability to reduce data size without compromising diagnostic accuracy or clinical relevance. Compression of ECG signals can optimize resource utilization, enhance transmission efficiency, and ensure timely access to critical cardiac information. Moreover, compressed ECG data enables seamless integration with existing healthcare infrastructure, facilitating real-time monitoring, teleconsultation, and telemedicine services even in bandwidth-constrained environments.

In recent years, researchers have explored various approaches for ECG signal compression [3], ranging from traditional methods based on signal processing techniques to advanced algorithms leveraging machine learning and artificial intelligence. These approaches aim to strike a balance between compression efficiency, computational complexity, and signal fidelity which helps in preserving the relevant information. Despite this, there are some challenges, as ECG signals are inherently non-stationary signals with different amplitudes, frequencies, and morphologies. This manuscript introduces a comprehensive study on ECG signal compression using Discrete Wavelet Transform (DWT) and Singular Value Decomposition (SVD), integrated with peak detection algorithms. We aim to investigate the efficacy of this approach in achieving high compression ratios while preserving the diagnostic integrity of the ECG signal. The combined approach of DWT and SVD helps in better compression as it utilizes the multiscale decomposition factor of DWT for capturing both low- and high-frequency components. This is followed by SVD's matrix factorization to reduce data dimensionality, retaining the significant singular values. Moreover, by incorporating peak detection algorithms into the compression process, critical features such

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as QRS complexes can be accurately identified and retained, further improving diagnostic fidelity. Through experimental validation and comparative analysis, we demonstrate the potential of our proposed method to offer a robust solution for efficient ECG data management in clinical and telemedicine applications.

The subsequent sections of this work are structured in the following manner: Section 2 presents a detailed survey of conventional compression techniques for ECG signals highlighting key trends and challenges. Section 3 gives detailed steps which are adopted for the compression of the ECG signal, again under section 3 an overview of the IoT system for telemedicine which makes the delivery of healthcare more convenient and efficient by the networked network is presented. Section 4 discusses the results obtained along with evaluation metrics and performance benchmarks used to assess the effectiveness of compression techniques. Finally, section 5 concludes with a discussion of the paper with a summary of our findings.

Contextual analysis

To contextualize the currently proposed system within the existing body of research, a comprehensive literature survey has been carried out to review and analyze the relevant scholarly works in the field. In [4] the paper introduced a novel approach to compressing ECG data by utilizing a two-dimensional (2D) transform method that capitalizes on redundancies between adjacent heartbeats and samples. The method involves cutting and aligning heartbeat data sequences to form a 2D data array, which is then subjected to transform coding using the 2D discrete cosine transform (DCT). The authors in [5] explained a novel lightweight ECG compression model for medical IoT systems. They have employed a hybrid compressor design followed by Huffman encoding, making it ideal for implementation on resource-constrained embedded platforms. The research paper [6] introduced a method for compressing ECG signals using the DWT and optimization approaches. This method aims to find the best wavelet parameters and threshold levels for achieving optimal compression. The optimization procedure entails establishing parameters, such as threshold levels and initial design angles, to minimize the compression ratio (CR) and Percent Root Mean Square Difference (PRD). Optimization algorithms such as Particle Swarm Optimization (PSO), Artificial Bee Colony, Cuckoo Search, and Firefly Algorithm are used to discover the most optimal values. Initially, the image undergoes Principal Component Analysis transformation, followed by decomposition using DWT, and further decomposition using canonical Huffman encoding was done in [7]. A one-level Haar wavelet transform was employed to break down 8-bit/24-bit key images with pixel sizes of 256×256 and 512×512 . The Canonical Huffman Coding-Discrete Wavelet Transform (CHC-DWT) approach was applied in reverse to reconstruct the image to obtain approximate and detailed coefficients, which resulted in the reconstructed image.

The authors in [8] used an approach that uses an optimized measurement matrix initialized with Gaussian random values. This matrix is then repeatedly refined to compress the original biosignal, specifically ECG signals. The approach utilized the Daubechies-4 (Db4) wavelet as a foundational matrix for sparsely representing ECG signals. It focused on preserving coefficients with significant absolute values, determined by energy thresholds. The compressed signal was then reconstructed using an accelerated non-uniform minimization method which is particularly important for wearable devices that have limited energy resources and for applications involving remote monitoring. On the other hand, the approach in [9] is about creating a template bank, for QRS complexes and plateau regions followed by processing using integer DCT, to compress the ECG signal. The plateau region underwent processing with integer DCT, quantization, and grouping with a significance map for signal compactness. Both QRS and non-QRS areas are encoded using the arithmetic encoding method. Again the paper's approach in [10] involved a compression strategy that utilized SVD and Adaptive Scanning Wavelet Difference Reduction (ASWDR) techniques to compress ECG signals. The Wavelet Difference Reduction (WDR) technique in the paper included two distinct scanning methods: fixed scan and adaptive scan of wavelet coefficients. WDR, in conjunction with ASWDR, enhanced the performance efficiency of the SVD approach. When the ASWDR approach is paired with WDR it results in good compression. In addition, the research in [11] used SVD and Embedded Zero Tree Wavelet techniques to compress the ECG signal. The authors transformed the ECG signal into a 2D format which enabled the extraction of signal characteristics.

Another research study [12] used truncated SVD. The authors used three methods for data reconstruction: a collection of singular triplets, correlated beat information, and various parameters. The singular triplets from SVD were essential for reconstructing ECG waveforms. Furthermore, the beat information that was linked to it determined the duration of each ECG cycle, which helped in recovering the ECG period. Again the methodology in the paper [13] focuses on a hybrid data compression scheme for power reduction in IoT wireless sensors, particularly in the context of ECG data. The Fan algorithm was utilized for compression, where only the most stringent slopes and their projections were estimated and stored to simplify calculations and reduce implementation complexity. The methodology in [14,15] depicts the selection of beta wavelets as the mother wavelet and performing a 4-level DWT decomposition on the ECG signal, followed by truncating wavelet coefficients below a threshold using level thresholding. Signal compression was further achieved by encoding the truncated coefficients using Run-Length Encoding (RLE) to eliminate redundant data without loss of signal information. In previous studies [16,17],

researchers have employed Empirical Mode Decomposition (EMD) and Wavelet Transform (WT) together in one algorithm, using the Haar mother function. It is the most effective in terms of compression.

Materials And Methods

Compression method framework and IoT system for telemedicine

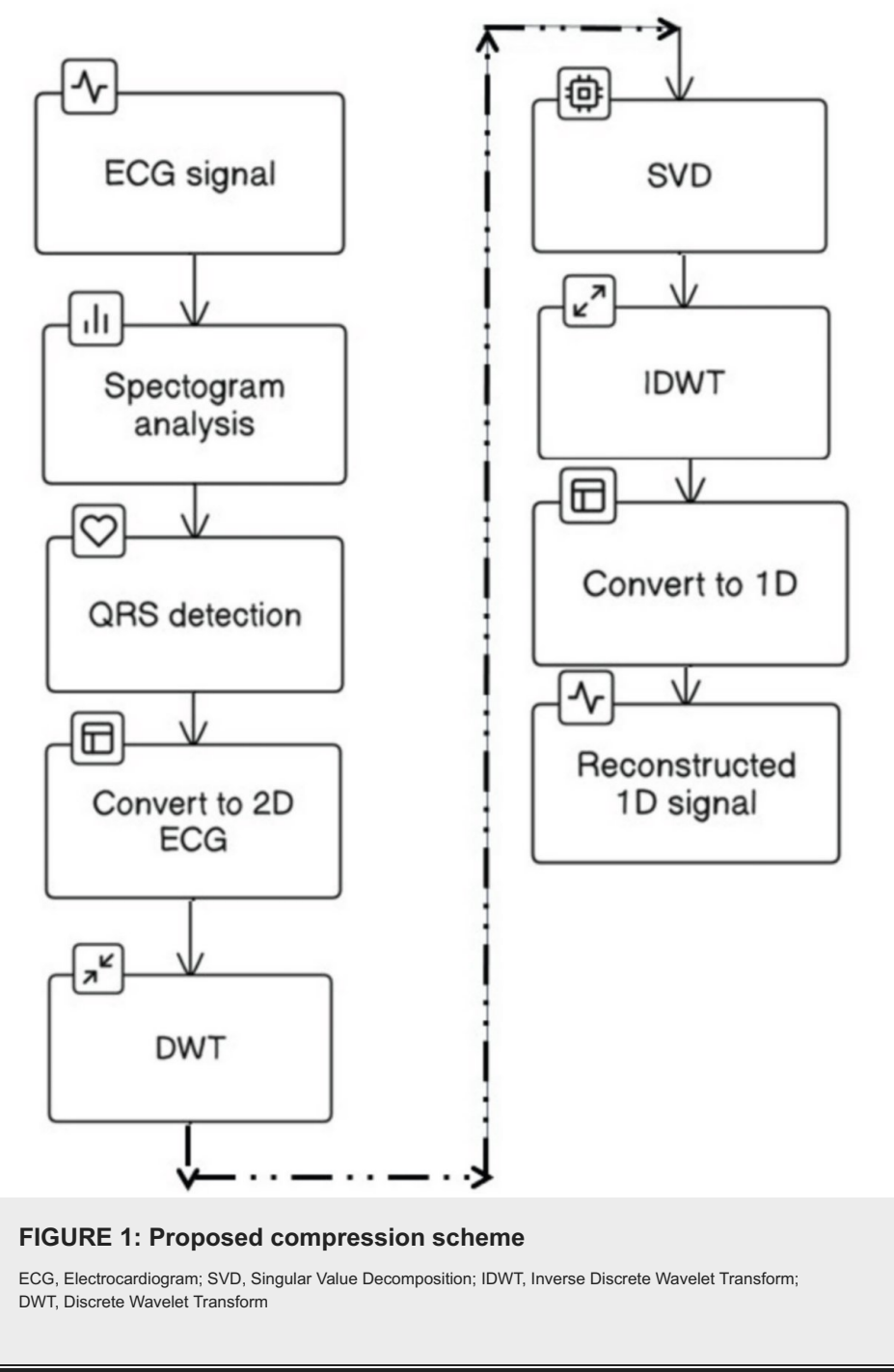
An approach to ECG compression for IoT healthcare systems is given.

1. Essential features are extracted from the ECG signal that captures its characteristics while reducing redundancy. Common features include R-peaks (the highest amplitude peaks), intervals between peaks (e.g., RR intervals), and waveform morphology descriptors.
2. Wavelet transform is used to decrease the dimensionality of the feature space while preserving the most significant information. This helps in reducing redundancy and computational complexity.
3. SVD is applied to further compress the quantized feature values.
4. Thoroughly validate the compression scheme using representative datasets and evaluate its performance in terms of compression ratio, reconstruction quality, computational complexity, and robustness to noise and artifacts.

The overall proposed system is depicted in Figure 1.

Proposed System for Compression

Through a detailed description of our methodology, we clarify the sequential steps involved in signal processing and peak detection and compression, laying the groundwork for subsequent experimental validation and performance assessment. Here using the Daubachies "db4" wavelet with a decomposition level of 3, we decompose the signal into approximation (LL) and detail (LH, HL, HH) sub-bands as mentioned below. The LL sub-band represents the approximation coefficients at the coarsest level of decomposition, capturing the low-frequency components of the ECG signals. On the other hand, the LH, HL, and HH sub-bands contain detailed coefficients corresponding to horizontal, vertical, and diagonal frequency components, respectively, highlighting high-frequency variations in the ECG signals. Daubachies wavelet is used due to the ability to analyze non-stationary signals like ECG and allowing efficient separation of low and high frequency. DB4 is particularly used in ECG compression as its waveform closely resembles the QRS complex components. The multi-level 2D wavelet decomposition process applied to $F(Y)$. The proposed compression scheme Figure 1 employs a sophisticated approach integrating DWT and SVD to efficiently handle ECG signals. Initially, the scheme converts the 1D ECG signal into a 2D format, enhancing spatial redundancy exploitation and facilitating clearer delineation of critical patterns such as QRS complexes. In our methodology, we begin by reading the ECG signals and their corresponding annotations using PhysioNet's WFDB toolbox (Waveform Database) in MATLAB 2023A (Matrix Laboratory). Specifically, we utilize the "rdann" function to read the annotations from the record file "MLII.dat" (Modified Limb Lead II), specifying "atr" as the annotation type and "1" to indicate the first annotation channel. Similarly, we employ the "rdsamp" function to read the ECG signals, specifying "MLII.dat" as the record file and "1" to indicate the first signal channel. ECG signal records were taken from 100 to 234 each of 1-minute duration of lead MLII. A sample ECG signal MLII reading can be found in Figure 2. Additionally, we visualize the first 4000 samples of the ECG signal to provide a closer examination of its characteristics, overlaying the annotated points. We then plot the entire ECG signal with annotations, highlighting points corresponding to cardiac events like R-peaks after R peak detection. Additionally, we visualize the first 4000 samples of the ECG signal, for closer examination in Figure 3. This visualization helps understand the temporal dynamics of the ECG signal, which is crucial for subsequent processing steps, hence the first 800 samples with detected R peaks in ECG record no. 106 as depicted in Figure 4. In our methodology, continuing to enhance the signal quality and extract relevant features, we employ a series of signal processing techniques, as depicted in Figure 5.



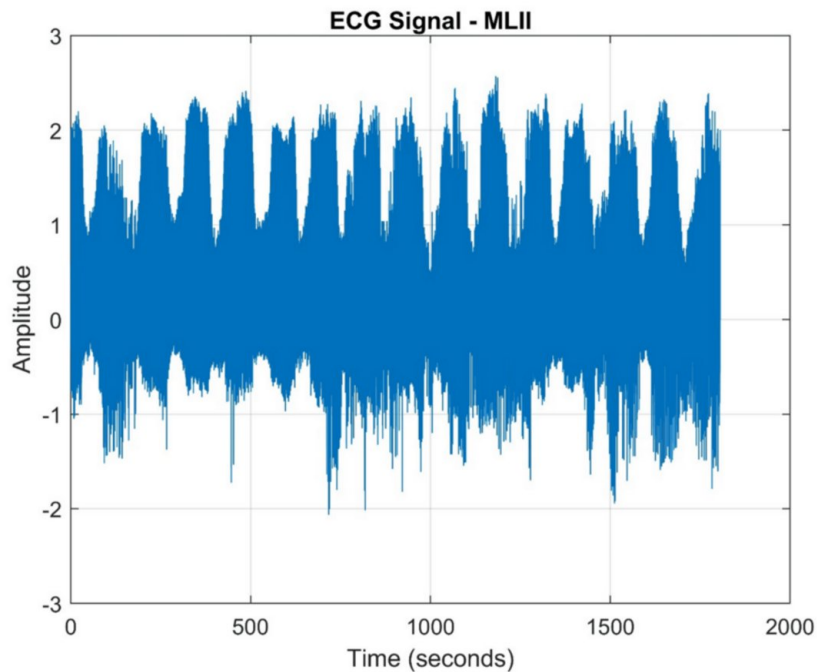


FIGURE 2: ECG signal lead MLII

ECG, Electrocardiogram

Considering the bulk of ECG signal MLII be $x(n)$ of length $(Q = M * N)$. So now arranging as a 1D signal, it will have a vector of length L . This vector contains the values of the ECG in a sequential manner. So if it is represented by $x(n)$ where $L = 0,1,2,...,L-1$ arranging it into a 1D signal would result in a vector x of length L such that:



Following data acquisition, we visualize the ECG signals and their annotations using MATLAB's plotting functions. First, we plot the entire ECG signal alongside its annotations, highlighting the points corresponding to annotated cardiac events such as R-peaks.

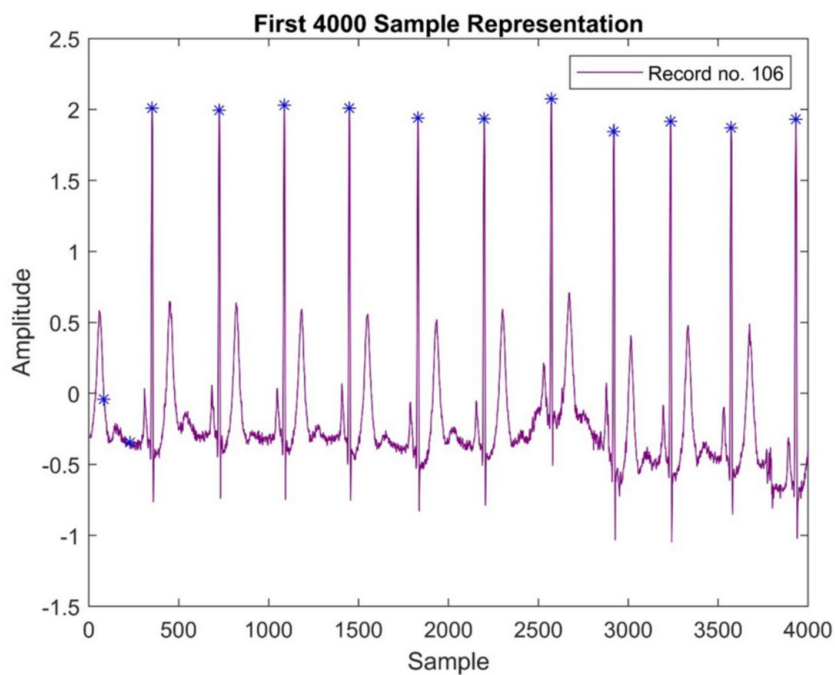


FIGURE 3: Detected R peaks in ECG record no. 106, first 4000 samples

ECG, Electrocardiogram

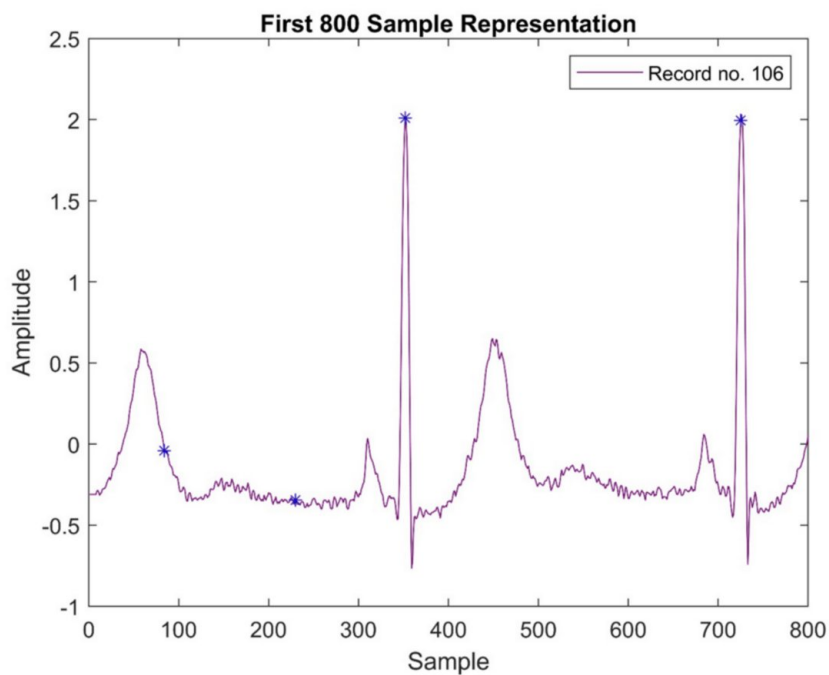


FIGURE 4: First 800 samples with detected R peaks in ECG record no. 106

ECG, Electrocardiogram

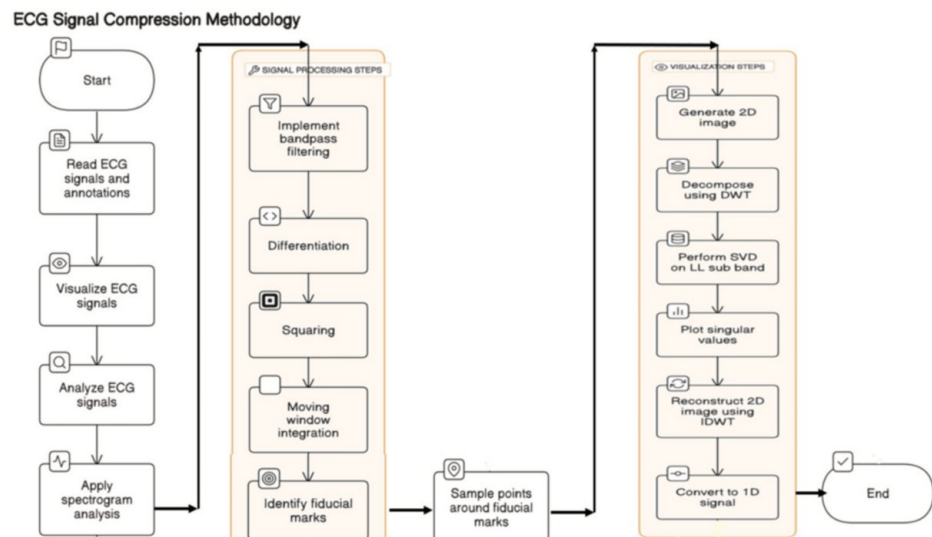


FIGURE 5: ECG signal compression methodology

ECG, Electrocardiogram; SVD, Singular Value Decomposition; IDWT, Inverse Discrete Wavelet Transform; DWT, Discrete Wavelet Transform

Now here x is our 1D signal of vector length L . Now we can reshape or convert it into a matrix with Y dimensions, ensuring no loss of data. So can be represented as:

$$Y = \begin{bmatrix} x(0) & x(1) & x(2) & \cdots & x(h+1) \\ x(h) & x(h+1) & x(h+2) & \cdots & x(2h-1) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x((g-1)h) & x((g-1)h+1) & x((g-1)h+2) & \cdots & x(L-1) \end{bmatrix}$$

Here, each element $Y(a, b)$ of the matrix Y , corresponds to the $X(q)$, where $\left\lfloor \frac{a}{h} \right\rfloor$ for $a = 0, 1, 2, \dots, g-1$ and $b = 0, 1, 2, \dots, h-1$. This arrangement ensures that the elements of the 1D X signal are re-arranged into 2D matrix in a column-major order. After this, we implement bandpass filtering with a bandpass filter kernel $h[n]$, to remove noise and isolate the frequency band of interest in the ECG signals. The filtered signal $Y_{filtered}[a, b]$ are then subjected to differentiation to highlight sharp waveform transitions by taking the finite difference of neighboring points in the 2D signal, followed by squaring to emphasize signal peaks and suppress baseline fluctuations. Subsequently, we apply moving window integration with a moving window function $w[a, b]$ to smoothen the signal and accentuate peak features.

$$Y_{integrated}[a, b] = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Y[m, n] H[a-m, b-n] (3) Y_{differentiated}[a, b] = Y_{integrated}[a+1, b] - Y_{integrated}[a, b] (4) Y_{squared}[a, b] = (Y_{differentiated}[a, b])^2 (5) Y_{compressed}[a, b] = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Y_{squared}[a+m, b+n] w[m, n] (6)$$

Here $Y_{squared}$ is the squared signal, is the integrated signal.

The preprocessing of the ECG signal begins with the modified Pan-Tompkins. The method includes bandpass filtering to remove baseline wander and high-frequency noise. The filtered signal is then differentiated to highlight the slopes of the QRS complex. After that squaring helps enhance the peaks making the QRS complexes more prominent. A moving window integration is applied to obtain a smoothed waveform suitable for thresholding. The threshold is adapted according to the accuracy of the peak detection. Finally, the QRS complexes are detected by applying an adaptive threshold. To transform the 1D ECG signal into a 2D representation, we have used the “reshape” function which converted the 1D vector to 2D matrix format.

Furthermore, fiducial marks corresponding to significant points in the ECG waveform, such as R-peaks, are identified using a peak detection algorithm applied to the integrated signal. These fiducial marks serve as reference points for subsequent analysis. Here $F(y)$ is the 1D array representing or containing the positions of the detected fiducial marks in the $Y_{integrated}$. To visualize the ECG signals in a comprehensive manner, we represented a 2D image by sampling points around each fiducial mark. This approach enables the creation of a structured representation of the ECG signals, facilitating the identification of temporal patterns and abnormalities. The detected peaks can be verified at Figure 4 and

Figure 6. Figure 4 speaks about the analysis of the sampled data within the first 800 samples of record no. 106. It focuses on the amplitude variations and identifying R peaks. Notably, there are two prominent peaks observed at approximately sample 350 and sample 750. This elucidates that we have observed characteristics and significant patterns in the dataset. The resulting spectrogram of Figure 7 provides a detailed time-frequency representation of the ECG signal. The X-axis provides time in seconds where Y-axis presents the frequency in Hz. The color intensity indicates the power spectral density in decibels (dB), which highlights the varying frequency components of the ECG signal over time. Similarly the sample point around each fiducial points can be seen in Figure 8. Additionally, we generate a 2D surface plot of the sampled points, providing a 3D perspective of the ECG waveform. By visualizing the signal in this manner, we gain insights into its amplitude variations over time, enhancing our understanding of its underlying dynamics. The representations are marked in Figure 9. These processed signals serve as inputs for subsequent stages of our methodology, including compression and tasks, contributing to the overall efficacy of our proposed approach.

$$F(s, t) = \text{peakdetection}(Y_{\text{integrated}})(7)$$

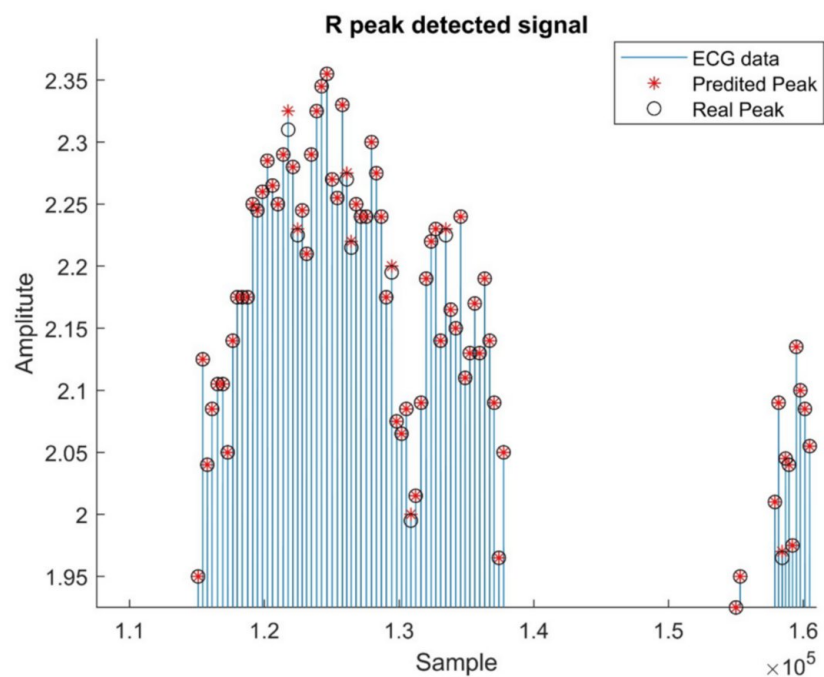


FIGURE 6: MLII with predicted and real R peaks

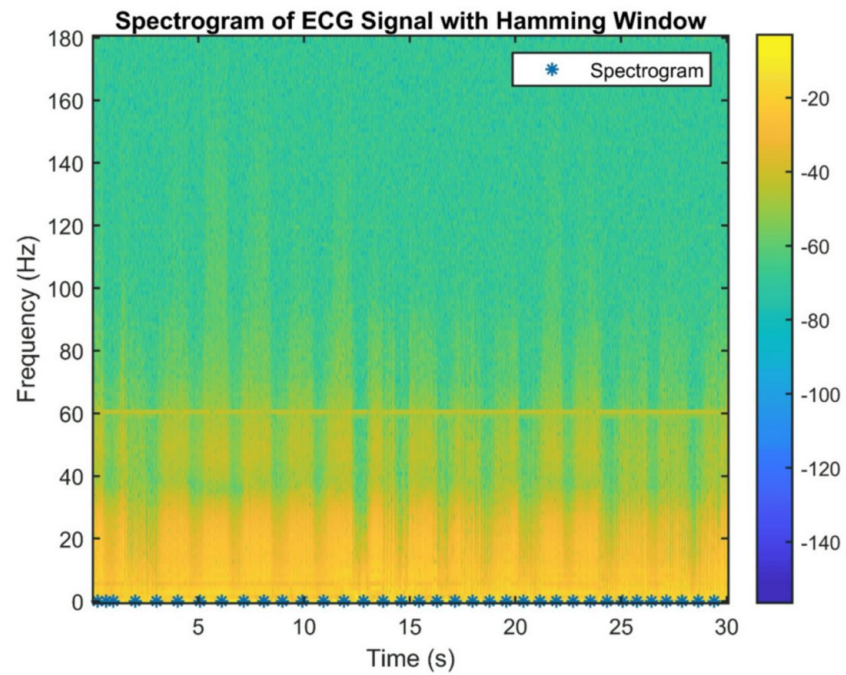
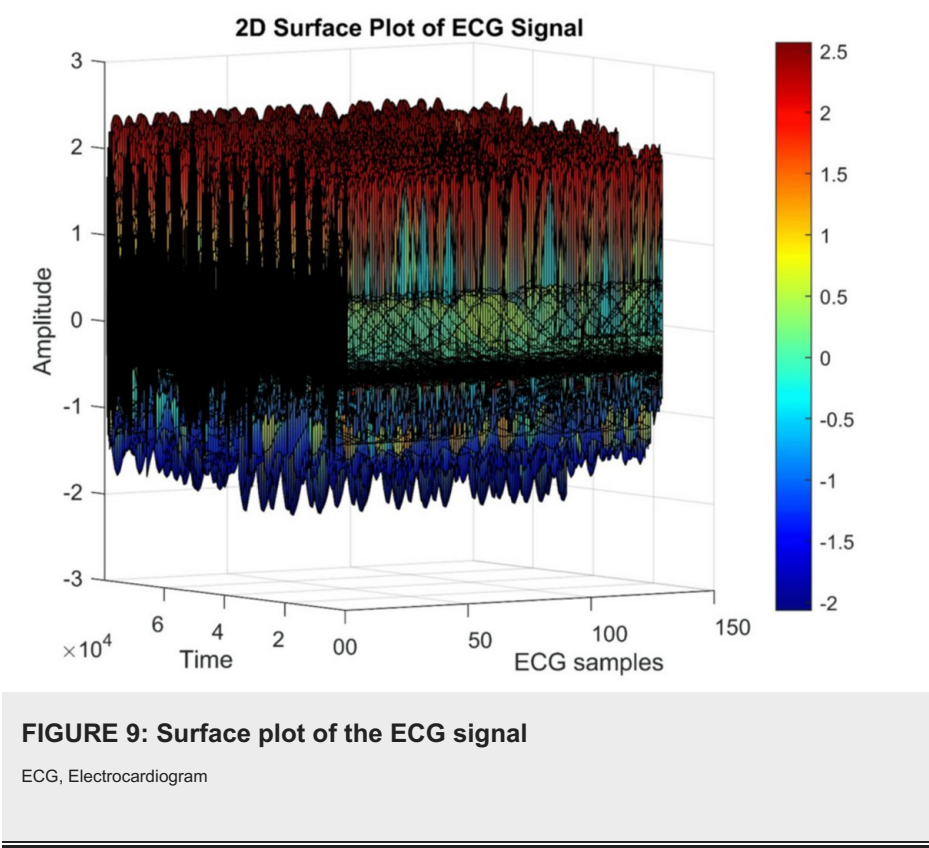
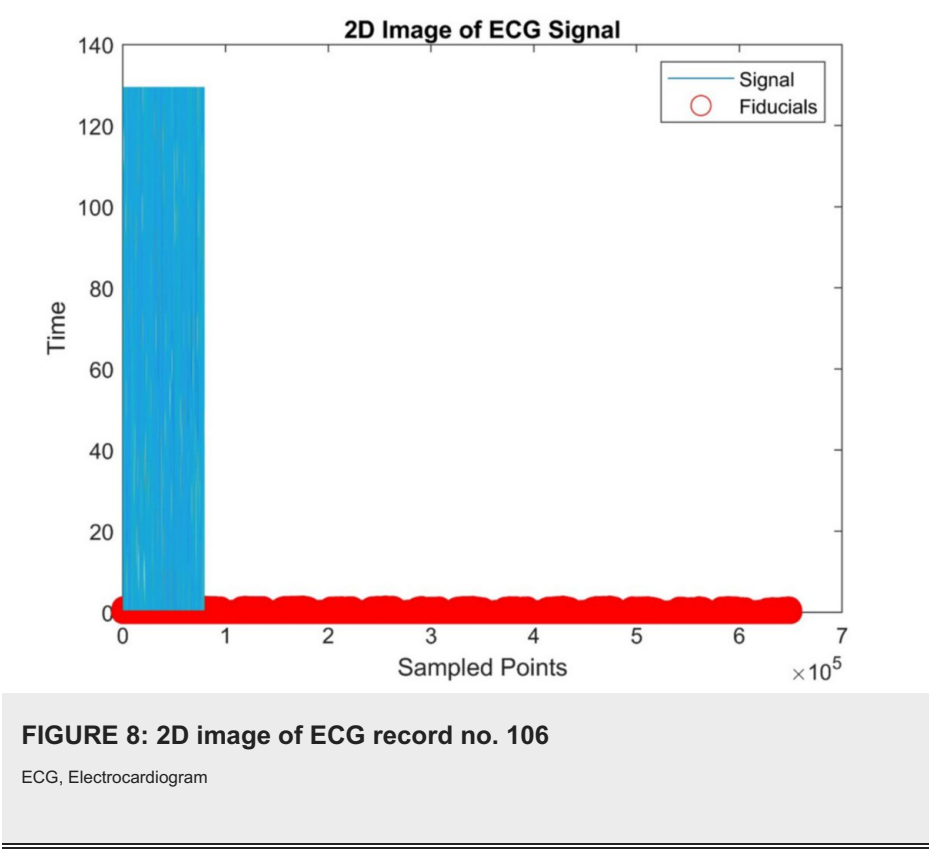


FIGURE 7: Spectrogram analysis of the ECG signal record no. 106

ECG, Electrocardiogram

Later we employed the DWT to decompose the sampled points obtained from the ECG signals into frequency sub-bands. The DWT splits the signal into multiple functions using the property of translation and dilation of a single function called as mother wavelet, and subsequently at each step of decomposition two parts grow as dilation and translation representing a low pass and high pass filter. But here in the case of 2D DWT analysis, $F(a,b)$ is the set of fiducial marks that are used for DWT analysis giving rise to 2^2 decomposed sub-bands i.e. three wavelet function $\Psi^h(a,b)$, $\Psi^v(a,b)$, $\Psi^d(a,b)$ and a scaling function $\Psi(a,b)$.



Here using the Daubachies "db4" wavelet with a decomposition level of 3, we decompose $F(a,b)$ the approximation (LL) and detail (LH, HL, HH) sub-bands as mentioned below. The LL sub-band represents the approximation coefficients at the coarsest level of decomposition, capturing the low-frequency components of the ECG signals. On the other hand, the LH, HL, and HH sub-bands contain detailed coefficients corresponding to horizontal, vertical, and diagonal frequency components,

respectively, highlighting high-frequency variations in the ECG signals. Figure 10 illustrates a multi-level 2D wavelet decomposition process applied to $F(Y)$.

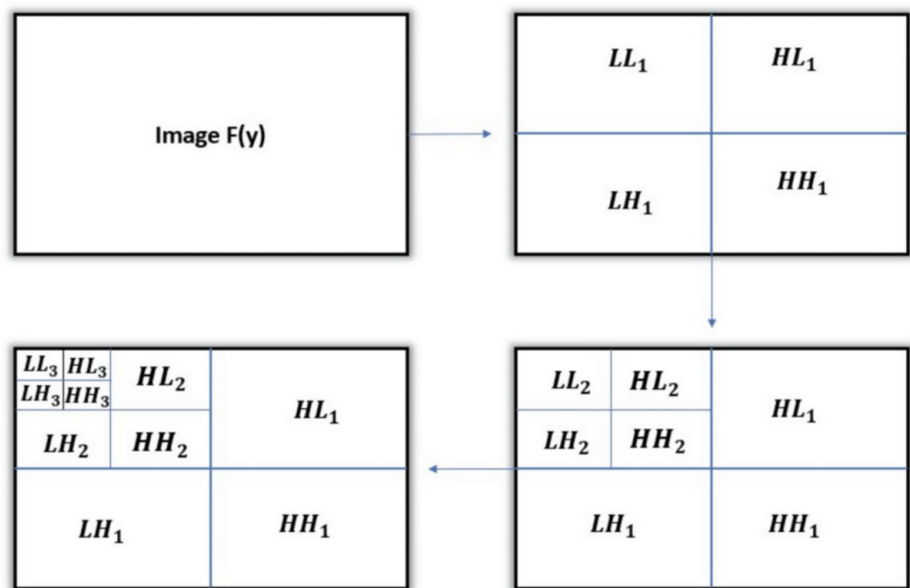


FIGURE 10: Three level decomposition by DWT

DWT, Discrete Wavelet Transform

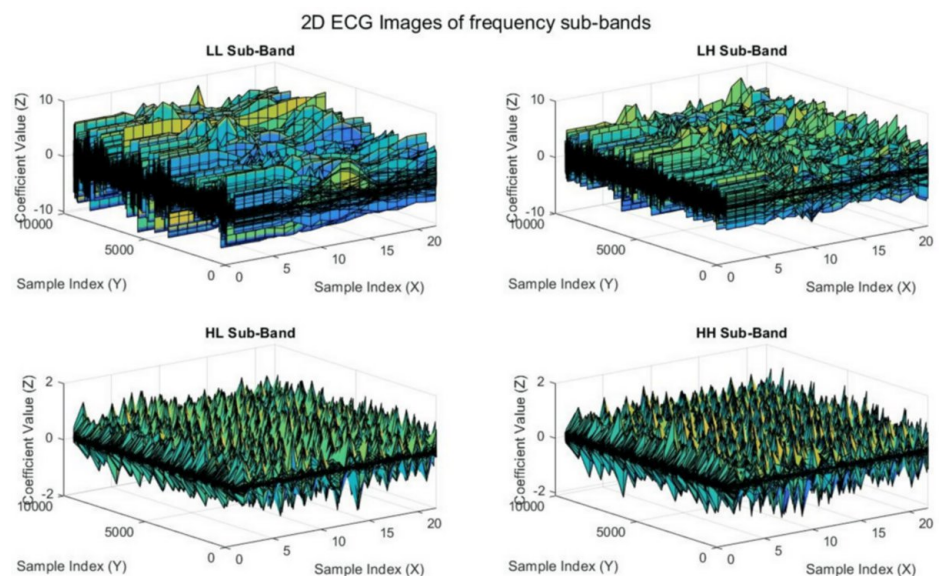


FIGURE 11: 2D ECG image of frequency sub-bands

ECG, Electrocardiogram

Now, let $\varphi^{m1}(a, b)$ and $\Psi^{(h,v,d)}(a, b)$ are associated with 2D MRA analysis, which is

$$\varphi^{m1}(a, b) = \varphi(a) \cdot \varphi(b) \quad (8)$$

$$\Psi^h(a, b) = \varphi(a) \cdot \Psi(b) \quad (9)$$

$$\Psi^v(a, b) = \varphi(b) \cdot \Psi(a) \quad (10)$$

$$\Psi^d(a, b) = \Psi(a) \cdot \Psi(b) \quad (11)$$

Here superscripts h,v,d are referring to the wavelet decomposition direction and $\varphi^{m1}(a, b) = LL$, $\Psi^h(a, b) = LH$, $\Psi^v(a, b) = HL$, $\Psi^d(a, b) = HH$. Now the multiresolution representation of the wavelet and scaling functions for the 2-dimension is given as :

$$\varphi_{(j,m,n)}(a, b) = 2^{j/2} \varphi(2^j a - m, 2^j b - n) \quad (12)$$

$$\Psi_{(j,m,n)}^i(a, b) = 2^{j/2} \Psi^i(2^j a - m, 2^j b - n) \quad (13) \quad \text{where } i \in \{h, v, d\}$$

Now the DWT function F(a,b) of size MXN is given as the scaling function is given as:

$$W_{\varphi}(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{a=0}^{M-1} \sum_{b=0}^{N-1} F(a, b) \varphi_{(j,m,n)}(a, b) \quad (14)$$

Hence the corresponding horizontal, vertical and diagonal representations of the converted image will be, the horizontal subband will be represented as:

$$W_{\Psi}^h(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{a=0}^{M-1} \sum_{b=0}^{N-1} F(a, b) \Psi_{(j,m,n)}^h(a, b) \quad (15)$$

The vertical subband will be represented as:

$$W_{\Psi}^v(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{a=0}^{M-1} \sum_{b=0}^{N-1} F(a, b) \Psi_{(j,m,n)}^v(a, b) \quad (16)$$

The diagonal subband will be represented as:

$$W_{\Psi}^d(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{a=0}^{M-1} \sum_{b=0}^{N-1} F(a, b) \Psi_{(j,m,n)}^d(a, b) \quad (16.1)$$

Hence the DWT of a 2D signal or image is given by:

$$F_{\Psi}^I(j, m, n) = \frac{1}{\sqrt{MN}} \sum_{a=0}^{M-1} \sum_{b=0}^{N-1} F(a, b) \Psi_{(j,m,n)}^I(a, b) \quad \text{where } I \in \{h, v, d\} \quad (17)$$

To visualize the frequency sub-bands obtained from the DWT decomposition, we generate 2D surface plots for each sub-band. These plots provide insights into the frequency distribution and spatial characteristics of the ECG signals across different scales and orientations. The presented methodology enables the decomposition of ECG signals into frequency sub-bands using DWT, facilitating the analysis of signal components at varying resolutions. This decomposition serves as a preprocessing step for further feature extraction and classification tasks, contributing to the comprehensive analysis of ECG data and the development of robust diagnostic tools. Figure 11 generated showcases the 2D surface plots for each frequency sub-band, providing a comprehensive representation of the frequency content of the ECG signals. This visualization aids in the interpretation of signal characteristics and the identification of relevant features for subsequent analysis and interpretation. Later we performed Singular Value Decomposition (SVD) on the approximation sub-band (LL) obtained from the Discrete Wavelet Transform (DWT) decomposition. The SVD decomposition yields three matrices: U, S, and V, Where U is left singular orthogonal vectors i.e. U(MXM) and V is the right singular orthogonal vectors i.e. V(NXN) and the diagonal matrix S is containing the singular values.

So now the decomposed LL sub-band let be E is subjected to SVD so the equation will be:

$$E = USV^T \quad (18)$$

The diagonal elements of S are the singular values of E, which is:

$$S = [Diag\{\tau_1, \tau_2, \tau_3, \dots, \tau_z\}] \quad (19)$$

Where $z=\min(M,N)$ such that:

$$\tau_1 \geq \tau_2 \geq \tau_3 \cdots \geq \tau_z \geq 0 \quad \text{and} \quad \tau_{z+1} = \tau_{z+2} = \cdots = \tau_n = 0 \quad (20)$$

The values of τu_1 are the singular values of E and are non-negative. So the i^{th} singular triplet is the set of $\{u_i, \tau_i, v_i\}$. Now the eigenvectors and the eigenvalues of $E^T E$ is:

$$E^T E = [U S V^T][U S V^T] = V S^T U^T U S V^T = V S^2 V^T \quad (21)$$

Similarly

$$E E^T = [U S V^T][U S V^T] = U [S V^T V S^T] U^T = U S^2 U^T \quad (22)$$

Here columns of $V = \text{eigenvectors of } E^T E$ and columns of $U = \text{eigenvectors of } E E^T$ and the eigenvalues are S^2 . Here the ECG signal is approximated by using far fewer elements by transforming E(MXN) into $U S V^T$.

$$\text{So if, } U = [u_1, u_2, u_3, \dots, u_M] \quad \text{and} \quad V = [v_1, v_2, v_3, \dots, v_N] \quad (23)$$

Now expanding the equation (18), we get:

$$E = \sum_{k=1}^z \tau_k u_k v_k^T = \tau_1 u_1 v_1^T + \tau_2 u_2 v_2^T + \cdots + \tau_r u_r v_r^T \quad (24)$$

To analyze the significance of the singular values, we plot them in descending order in SVD analysis of LL Subband. This plot provides insights into the energy distribution across the singular vectors and helps identify dominant modes of variation in the approximation sub-band. By examining the decay behaviour of the singular values, we can determine the intrinsic dimensionality of the LL sub-band and assess its compression potential. The SVD decomposition serves as a dimensionality reduction technique, enabling the representation of the approximation sub-band in a lower-dimensional space while preserving its essential characteristics. This dimensionality reduction facilitates efficient data representation and compression, making it an integral part of our proposed methodology for ECG signal analysis and compression. Following the SVD process, we reconstruct the original 2D image using the Inverse Discrete Wavelet Transform (IDWT), aiming to preserve the essential characteristics of the ECG signals while achieving data compression.

$$E(M, N) = \frac{1}{\sqrt{MN}} \sum_m \sum_n [E_\varphi(j_0, m, n) \varphi_{j_0, a, b}(m, n)] + \frac{1}{\sqrt{MN}} \sum_{i=h, v, d} \sum_{j=j_0} \sum_m \sum_n [E_\psi^i(j, a, b) \Psi_{j, a, b}^i(m, n)] \quad (25)$$

Visualizing the reconstructed 2D image provides knowledge of the fidelity of the compression process and the preservation of diagnostic information. Finally, again the signal is converted to 1-dimensional for visual accuracy. Additionally, we incorporate signal processing functions for bandpass filtering and fiducial detection, enhancing the robustness and accuracy of the compression process. The bandpass filter effectively removes noise and isolates relevant frequency components, while the fiducial detection algorithm identifies key cardiac events, ensuring the preservation of clinically significant features in the compressed ECG.

IoT system for telemedicine

With the advancements, in technology the possibilities of IoT in the healthcare sector are expanding [18]. The progress in telemedicine and remote patient monitoring is enhancing the provision of healthcare services ensuring they are more convenient and effective for individuals seeking care.

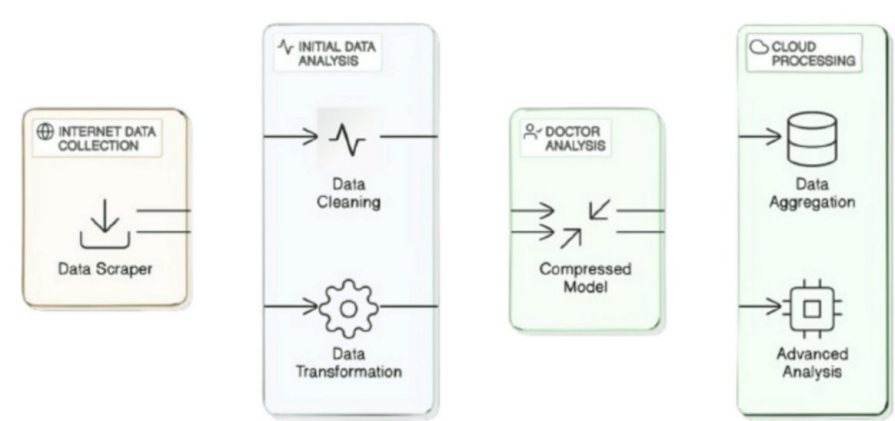


FIGURE 12: Data processing and analysis flow for Medical data

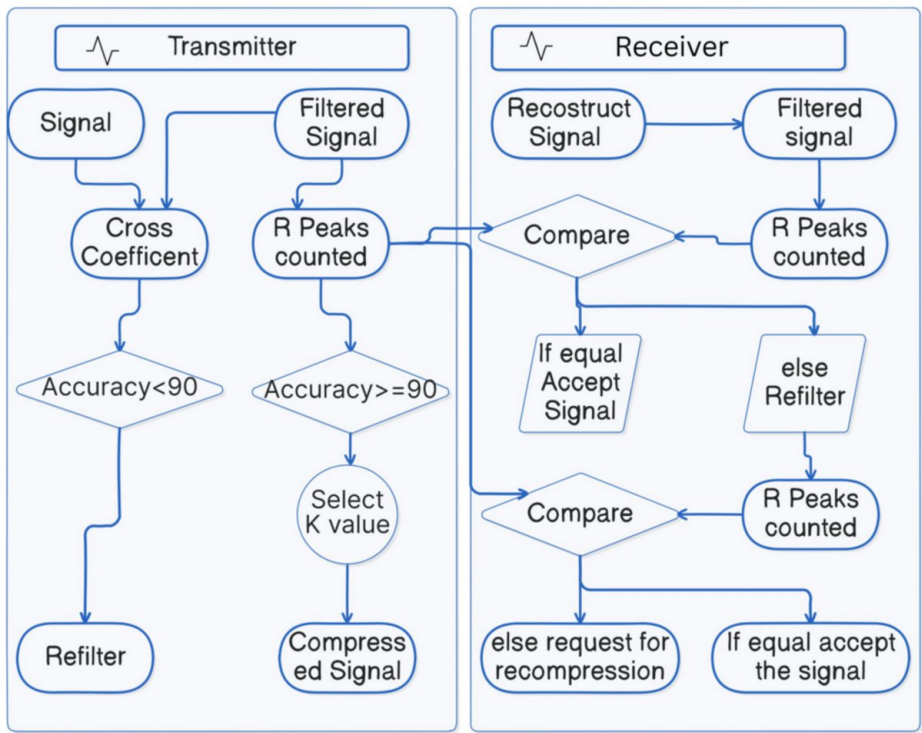


FIGURE 13: Flowgraph of proposed work by transmitter and receiver

In Figure 12, the journey of ECG data begins from the data scraper i.e. gathering the various ECG data from sources. This raw data then undergoes initial analysis. Next, the data are standardized to a specific format and prepared for further processing and ensuring compatibility across different platforms and systems. Then, the compressed model is utilized by the doctor for efficient data interpretation. This is followed by transmitting the data to the cloud for centralized processing and aggregation. Similarly, Figure 13 depicts the signal processing workflow diagram explaining the transmitter and receiver. The transmitter transmits the signal to the receiver, which undergoes filtering; subsequently, the proposed algorithm is used to identify the R peaks, ensuring accurate cardiac event detection. Based on the accuracy of the peak detection, the system dynamically adjusts threshold (k) to optimize signal processing. Further downstream, the proposed compression techniques are employed to reduce data size without compromising diagnostic integrity. Finally, the compressed signal is reconstructed maintaining fidelity and presented for analysis, assisting in efficient medical interpretation and decision-making.

The two-level compression technique WT and SVD ensures that the ECG data is efficiently encoded, which reduces the transmission time and bandwidth requirements. Compressed signals take up less space reducing network load. It also reduces the computational load. The combination of DWT and SVD captures both high- and low-frequency components of the ECG signal while removing noise which leads to better analysis in remote monitoring.

Results

Results and evaluation metrics with performance benchmarks

Curated Dataset

The dataset is curated from the MIT-BIH (Massachusetts Institute of Technology-Boston's Beth Israel Hospital) Physionet. It is one of the three globally recognized ECG databases and has been utilized for training the proposed algorithm. The dataset is comprised of 100 to 234 records of mitdb and the lead used is MLI commonly used for monitoring various cardiac conditions and abnormalities. Within the dataset, all the recorded signals are at a sampling rate of 360 Hz and possess an 11-bit resolution.

Evaluation Metrics

Furthermore, we investigate the performance of the compression technique by assessing various metrics, including CR, PRD, Quality Score (QS), Signal_to_Noise Ratio (SNR), and Peak Signal_to_Noise Ratio (PSNR). These metrics offer quantitative measures of the compression efficacy, providing valuable insights into the trade-offs between data compression and signal fidelity.

1. Compression Ratio (CR):

$$CR = \frac{\text{Size of the Original signal in bits}}{\text{Size of the compressed signal in bits}}$$

A higher CR % indicates a greater reduction in data size through compression.

2. Percent_root_mean_square_difference (PRD):

$$PRD = \sqrt{\frac{\sum (O(n) - R(n))^2}{\sum O(n)^2}}$$

Where, $O(n)$ = OriginalSignal # $R(n)$ = Reconstructedsignal

3. Quality Score (QS):

Excellent scores of high-quality score indicates good compression performance.

$$QS = \frac{CR}{PRD}$$

4. Signal-to-Noise Ratio (SNR):

It gives the idea of noise energy added to a signal due to compression and decompression.

$$SNR = 10 \log \left(\frac{\sum (O(n) - \overline{O(n)})^2}{\sum (O(n) - R(n))^2} \right)$$

$O(\overline{n})$ = Mean value of $O(n)$

$O(n)$ = Original Signal

$R(n)$ = Reconstructed Signal

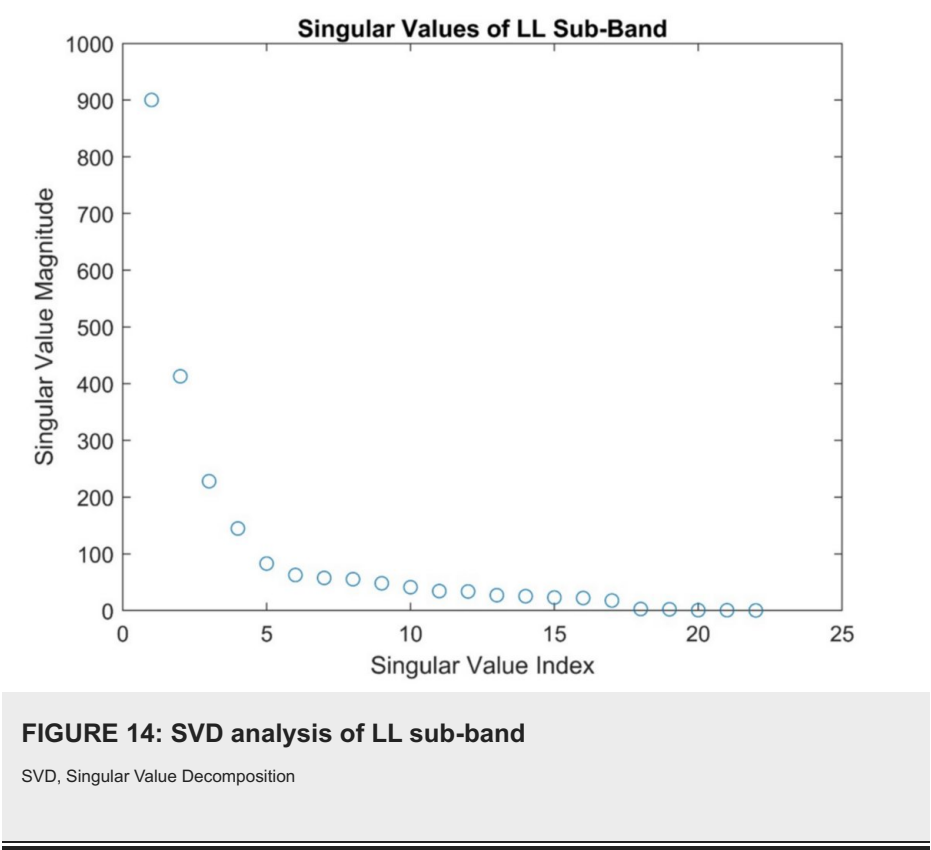
5. Peak Signal-to-Noise Ratio (PSNR):

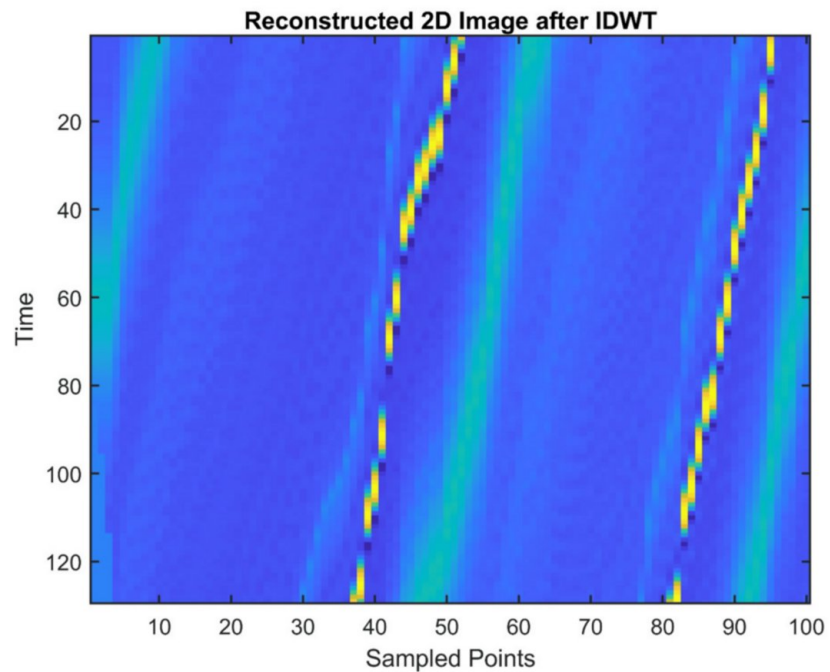
It quantifies the ECG signal. A good and higher PSNR ensures that critical medical information remains accurate and reliable for diagnostic purposes.

$$\text{PSNR} = 20 \log_{10} \left(\frac{\max(x(n))}{\sqrt{\text{MSE}}} \right)$$

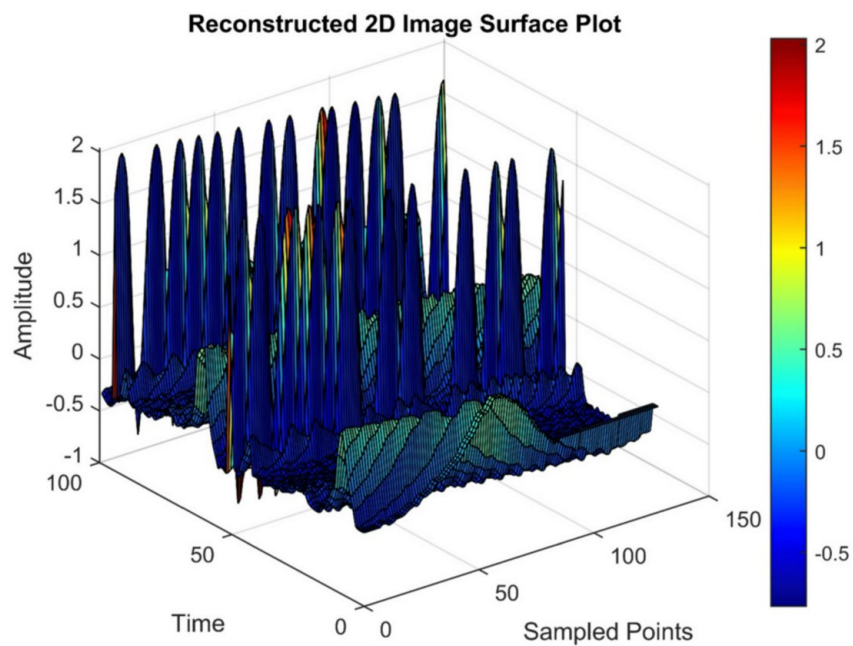
The results presented in this section provide an analysis of all the outputs obtained from the proposed system.

The SVD analysis of LL Subband (Figure 14) generated plot of singular values gives information about structure and variability of approximation sub-band, guiding the selection of relevant features and compression strategies. This analysis aids in the design of effective compression algorithms tailored to the characteristics of ECG signals, ensuring optimal utilization of resources and preservation of diagnostic information.



**FIGURE 15: Reconstructed 2D image after IDWT**

IDWT, Inverse Discrete Wavelet Transform

**FIGURE 16: Reconstructed 2D image and surface plot**

Similarly, Figure 15 reveals a series of diagonal bands characterized by varying intensity levels, indicated by the color spectrum from blue to yellow. These suggest the presence of periodic components in the original signal, which are effectively reconstructed through the IDWT process. Figure 16 provides a comprehensive visualization of the reconstructed signal's amplitude variations, which facilitates the identification of key features and periodic patterns. It highlights the robustness of the IDWT in preserving the features of the original signal. The sharp, high amplitude peaks indicate the presence of high-frequency components corresponding to the QRS complex. The smoother regions in the plot represent low

amplitude components corresponding to baseline wander or T-waves. The amplitude variations suggest minimal distortion, which indicates the reconstructed signal closely resembles the original ECG waveform. In Figure 17 we found there is a high visual similarity between the two plots indicates that the reconstruction process preserves signal fidelity. There is minimal observable distortion or noise, implying that the method used for reconstruction is highly effective given the PRD and CR values.

In Figure 18, the original signal MLII exhibits consistent amplitude variations, characterized by periodic fluctuations between -2 and 2. The reconstructed signal mirrors these variations with high precision, maintaining the periodicity and amplitude range observed in the original signal. Further validation is provided by the comparison of the first 800 sample representations and their reconstruction in Figure 18. Notably, peaks around the 300th, 400th, and 700th samples are preserved, indicating that the reconstruction process successfully captured essential characteristics of the original signal, while minor variations do not significantly impact the overall signal representation. Again the presented Figure 19 illustrates an IoT-enabled system for ECG signal acquisition, synchronization, and cloud-based analysis. For synchronization, Raspberry Pi 4 model B is used which supports ample processing power to handle data synchronization tasks. The smartphone acts as a connectivity bridge between Raspberry Pi and the cloud database. The cloud database serves as a centralized repository, which facilitates secure storage as well as efficient retrieval of data for subsequent analysis. Below we have provided the different performance metrics of the proposed method which suggests that the presented method is much more evident and effective than others having a CR of 98.82 with lower PRD as low as 1.2034. This can be seen in the comparison of Table 2, and Table 1 provides the proposed system’s performance metrics.

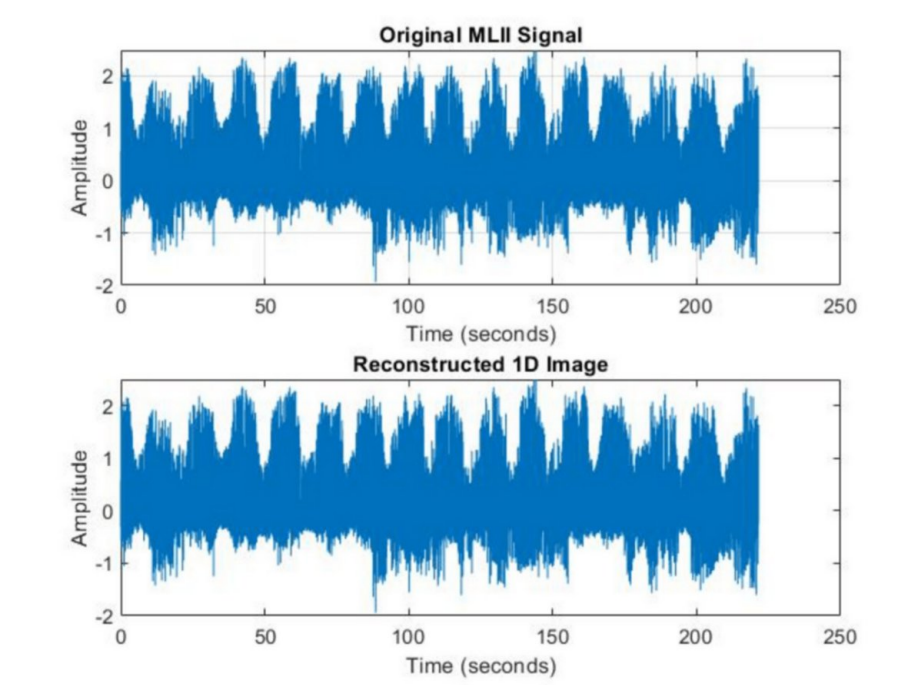


FIGURE 17: MLII input ECG signal versus the reconstructed signal of MLII

ECG, Electrocardiogram

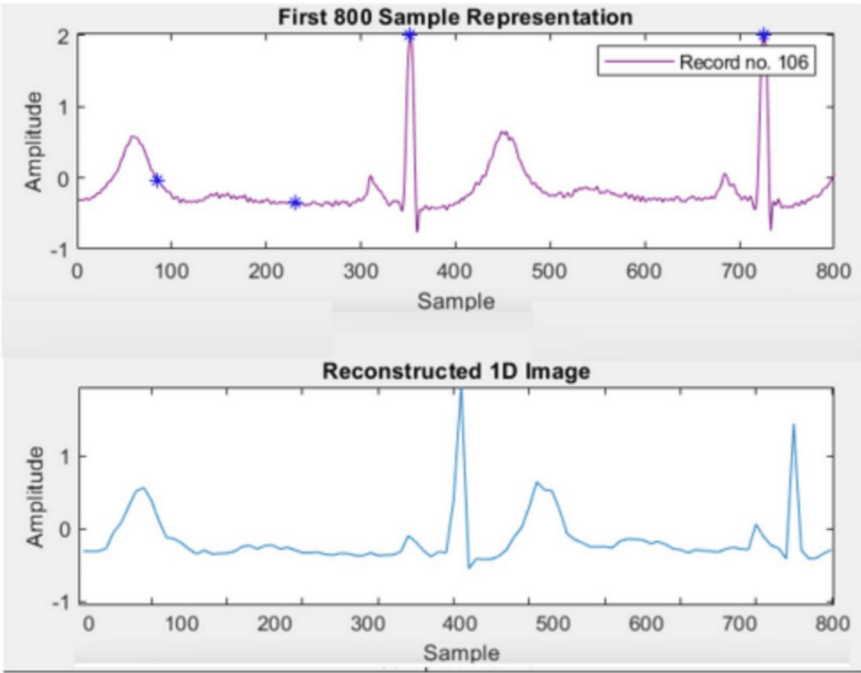


FIGURE 18: For better visualization input record no. 106 versus the reconstructed signal of record no. 106

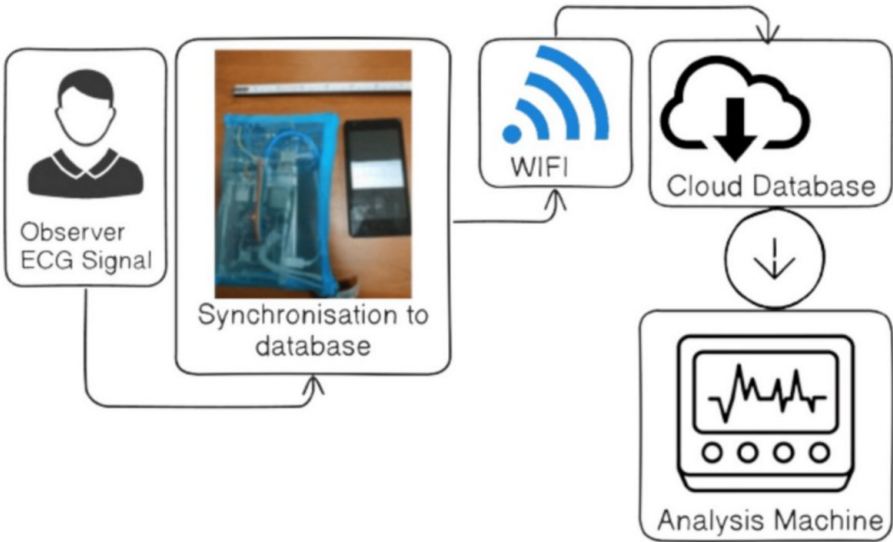


FIGURE 19: IoT enabled wireless upload to cloud for Medical Data

IoT, Internet of Things

Performance Metrics	Values
Compression Ratio (CR)	98.82
Percent-root-mean-square-difference (PRD)	1.2034
Quality Score (QS)	82.345
Signal-to-noise ratio (SNR)	4.1858
Peak Signal-to-Noise Ratio (PSNR)	1.8013

TABLE 1: Proposed system’s performance metrics

Reference	Year	Methodology	Dataset	CR (%)	PRD	QS	SNR	PSNR
[19]	2008	MLP- NN with LZW	24 records MIT-BIH Arrhythmia database	12.74:1	0.61	20.88	-	-
[20]	2020	RRO-NRDPWT	MIT-BIH Arrhythmia + PTB database	66.9	3% ± 0:05		-	-
[21]	2021	EMD + DWT + RLE	48 ECG records of MIT-BIH arrhythmia database	25.74	1.91	13.44		
[22]	2021	BWT + MTF + RLE	A testbed of 47 ECG records are MIT-BIH Arrhythmia database	6.34	0%	-	-	-
[23]	2021	B-Spline Interpolation + AntColony Optimization	48 records from MIT-BIH Arrhythmia database	87.1	2.3	3.4	-	-
[24]	2022	Sparsity enhancing wavelets design	MIT challenge data set A (Challenge 2013 Training set A)	41	2.0	20.50		
[25]	2023	Fractal Model + Iterated Function System	MIT-BIH Arrhythmia Database Record No. 119	30	0.102	-	-	-
[26]	2023	QRS complex segment bank + Integer DCT based Plateau region Processing	MIT-BIH Arrhythmia database	31.51	0.84	28.57	-	-
[27]	2024	PWM (Pulse Width Modulation) based	MIT-BIH Arrhythmia database	90.4	0.33	79.22	-	-
[28]	2024	DWT + DCT based compressed sensing	MIT-BIH Arrhythmia database	87.7	2.4	-	-	-
[29]	2024	FPGA enabled Adaptive Predictor	MIT-BIH PTB (Pulmonary Tuberculosis) Database	45.23	Lossless	-	-	-
Current Proposed Study	2024	Proposed Method	MIT-BIH Arrhythmia Database (100-234) MLII	98.82	1.2034	82.345	4.185	1.8013

TABLE 2: Proposed system’s performance metrics

DWT, Discrete Wavelet Transform; RLE, Run-Length Encoding; DCT, discrete cosine transform; MIT-BIH, Massachusetts Institute of Technology-Boston’s Beth Israel Hospital; ECG, Electrocardiogram; CR, Compression Ratio; PRD, Percent-root-mean-square-difference; QS, Quality Score; SNR, Signal-to-noise ratio; PSNR, Peak Signal-to-Noise Ratio

The comparative analysis table presents a detailed overview of various ECG signal compression techniques from 2008 to 2024, focusing on key performance metrics such as CR, PRD, and QS (Table 2). Our proposed multistage DWT-SVD method demonstrates superior performance, achieving the highest CR of 98.82% and relatively low PRD of 1.2034, indicating excellent signal fidelity. A low PRD indicates

minimal distortion between the original and reconstructed signal.

Discussion

The detailed pseudocode outlined below illustrates the comprehensive approach taken in this method, highlighting the key stages of fiducial point detection, wavelet decomposition, and reconstruction.

Pseudocode_1: Proposed Wavelet-based Compression for ECG Signal

```

· Initialization:

· Load ecg_samples and ann_samples.

· Set parameters: low_cutoff, high_cutoff, window_size.

· N = size(ecgsignals); b = N(1,2).

· Preprocessing:

· Bandpass Filter ecgsignals using Butterworth filter with cutoff frequencies low_cutoff and high_cutoff.
  For each i = 1 to b

· filtered_ecg = bandpass_filter(ecgsignals(:, i), low_cutoff, high_cutoff, Fs)

· Compute Derivative: derivatives = diff(filtered_ecg)

· Square the derivative signal: squared_signal = derivatives.^2

· Apply Moving Window Integration: integrated_signal = movsum(squared_signal, window_size) End For

· Fiducial Point Detection:

· For each i = 1 to b

· Detect fiducial points using findpeaks on integrated_signal: fiducial_marks =
  detect_fiducials(integrated_signal) End For

· 2D Image Creation:

· For each i = 1 to length(fiducial_marks)

· Extract 129 points around each fiducial mark to create 2D image matrix sampled_points.

End For

· Wavelet Decomposition:

· For each i = 1 to b

· Apply 2D Discrete Wavelet Transform (DWT): [C1, S] = wavedec2(sampled_points, level, wavelet)

· Extract wavelet sub-bands:

o LL = appcoef2(C1, S, wavelet, level)

o LH = detcoef2('h', C1, S, level)

o HL = detcoef2('v', C1, S, level)

o HH = detcoef2('d', C1, S, level)

End For

· Singular Value Decomposition (SVD):

```

- For each $i = 1$ to b
- Perform SVD on LL sub-band: $[U, S, V] = \text{svd}(LL)$ End For
- Image Reconstruction:
- For each $i = 1$ to b
- Reconstruct the 2D image using Inverse DWT: $\text{reconstructed_image_subset} = \text{waverec2}(C1, S, \text{wavelet})$
- Extract a specific row or column: $\text{reconstructed_1d} = \text{reconstructed_image_subset}(:, \text{selected_column})$
End For
- Metrics Calculation:
- Compression Ratio (CR): $N4 = b \times (O \times 3 + 2)$ $N4 = b \times (O \times 3 + 2)$ $CR = \frac{N2(1,1)}{N4}$ $CR = \frac{N2(1,1)}{N4}$
- Calculate other metrics (PRD, QS, SNR, PSNR) using appropriate formulas.
- Output:
- Store and display the reconstructed ECG signal reconstructed_1d and metrics CR, PRD, QS, SNR, PSNR.

Conclusions

This study demonstrates the efficacy of a two-level compression model combining the Wavelet transform with Daubechies followed by SVD for further data reduction. Our proposed compression framework achieves optimal compression performance which is 98.82 CR having only 1.2034 PRD while maintaining the diagnostic integrity of ECG signals. The results indicate that this approach significantly enhances the efficiency and reliability of IoT-based communication in healthcare settings. This can facilitate timely and accurate patient monitoring and care delivery. The proposed technique thus holds a strong hold on the scalability and responsiveness of telemedicine solutions. The technique can be extended to a broader set of bio signals such as PPG (photoplethysmography) and EEG (Electroencephalogram). Additionally, future work will focus on deploying this algorithm on real-time embedded platforms, considering energy and memory constraints. Specifically, it can be explored in ARM Cortex-M microcontrollers.

Additional Information

Author Contributions

All authors have reviewed the final version to be published and agreed to be accountable for all aspects of the work.

Concept and design: Sudeshna Baliarsingh, Mihir Narayan Mohanty

Acquisition, analysis, or interpretation of data: Sudeshna Baliarsingh, Mihir Narayan Mohanty

Drafting of the manuscript: Sudeshna Baliarsingh

Critical review of the manuscript for important intellectual content: Sudeshna Baliarsingh, Mihir Narayan Mohanty

Supervision: Mihir Narayan Mohanty

Disclosures

Human subjects: All authors have confirmed that this study did not involve human participants or tissue.

Animal subjects: All authors have confirmed that this study did not involve animal subjects or tissue.

Conflicts of interest: In compliance with the ICMJE uniform disclosure form, all authors declare the following: **Payment/services info:** All authors have declared that no financial support was received from any organization for the submitted work. **Financial relationships:** All authors have declared that they have no financial relationships at present or within the previous three years with any organizations that might have an interest in the submitted work. **Other relationships:** All authors have declared that there are no other relationships or activities that could appear to have influenced the submitted work.

References

1. Kesavan T, Kaliappan E, Sivaranjani S, Rameshkumar K, Kathirvel P: IoMT-based telemedicine monitoring machine for COVID patients. Internet of Medical Things in Smart Healthcare. Apple Academic Press, 2023. 1-10.
2. Kotsialou Z, Makris N, Gall S: Fundamentals of the electrocardiogram and common cardiac arrhythmias. Anaesthesia & Intensive Care Medicine. 2024, 25:219-22. [10.1016/j.mpaic.2023.11.014](https://doi.org/10.1016/j.mpaic.2023.11.014)
3. Jha CK, Kolekar MH: Electrocardiogram data compression techniques for cardiac healthcare systems: a methodological review. IRBM. 2022, 43:217-28. [10.1016/j.irbm.2021.06.007](https://doi.org/10.1016/j.irbm.2021.06.007)
4. Lee H, Buckley KM: ECG data compression using cut and align beats approach and 2-D transforms. IEEE Transactions on Biomedical Engineering. 1999, 46:556-64. [10.1109/10.759056](https://doi.org/10.1109/10.759056)
5. Chang Y, Sobelman GE: Lightweight Lossy/Lossless ECG compression for Medical IoT systems. IEEE Internet of Things Journal. 2024, 11:12450-458. [10.1109/jiot.2023.3336995](https://doi.org/10.1109/jiot.2023.3336995)
6. Pal HS, Kumar A, Vishwakarma A: TQWT based electrocardiogram compression using optimized thresholding. Advanced Communication Technologies and Signal Processing (ACTS). IEEE. 2021, 1-5. [10.1109/ACTS53447.2021.9708289](https://doi.org/10.1109/ACTS53447.2021.9708289)
7. Ranjan R, Kumar P: An improved image compression algorithm using 2D DWT and PCA with Canonical Huffman Encoding. Entropy. 2023, 25:1382. [10.3390/e25101382](https://doi.org/10.3390/e25101382)
8. Wu FY, Yang K, Sheng X: Optimized compression and recovery of electrocardiographic signal for IoT platform. Applied Soft Computing. 2020, 96:106659. [10.1016/j.asoc.2020.106659](https://doi.org/10.1016/j.asoc.2020.106659)
9. Pandey A, Saini BS, Singh B, Sood N: A 2D electrocardiogram data compression method using a sample entropy-based complexity sorting approach. Computers and Electrical Engineering. 2016, 56:30-45. [10.1016/j.compeleceng.2016.10.012](https://doi.org/10.1016/j.compeleceng.2016.10.012)
10. Kumar R, Kumar A, Singh GK: Electrocardiogram signal compression based on singular value decomposition (SVD) and adaptive scanning wavelet difference reduction (ASWDR) technique. AEU-International Journal of Electronics and Communications. 2015, 69:1810-22. [10.1016/j.aeue.2015.09.011](https://doi.org/10.1016/j.aeue.2015.09.011)
11. Kumar R, Kumar A, Singh GK: Hybrid method based on singular value decomposition and embedded zero tree wavelet technique for ECG signal compression. Computer Methods and Programs in Biomedicine. 2016, 129:135-48. [10.1016/j.cmpb.2016.01.006](https://doi.org/10.1016/j.cmpb.2016.01.006)
12. Wei JJ, Chang CJ, Chou NK, Jan GJ: ECG data compression using truncated singular value decomposition. IEEE Transactions on Information Technology in Biomedicine. 2001, 5:290-99. [10.1109/4233.966104](https://doi.org/10.1109/4233.966104)
13. Deepu CJ, Heng CH, Lian Y: A hybrid data compression scheme for power reduction in wireless sensors for IoT. IEEE Transactions on Biomedical Circuits and Systems. 2017, 11:245-54. [10.1109/tbcas.2016.2591923](https://doi.org/10.1109/tbcas.2016.2591923)
14. Kumar R, Kumar A, Pandey RK: Beta wavelet based ECG signal compression using lossless encoding with modified thresholding. Computers and Electrical Engineering. 2013, 39:130-40. [10.1016/j.compeleceng.2012.04.008](https://doi.org/10.1016/j.compeleceng.2012.04.008)
15. Baliarsingh S, Panda PK, Mohanty MN: Unsupervised technique for cardiac data compression with β wavelet transform for telemedicine. Measurement: Sensors. 2023, 26:100671. [10.1016/j.measen.2023.100671](https://doi.org/10.1016/j.measen.2023.100671)
16. Chandra S, Sharma A, Singh GK: Computationally efficient cosine modulated filter bank design for ECG signal compression. IRBM. 2020, 41:2-17. [10.1016/j.irbm.2019.06.002](https://doi.org/10.1016/j.irbm.2019.06.002)
17. Baliarsingh S, Panda PK: An optimal transform based technique for ECG signal compression. Journal of Information and Optimization Sciences. 2022, 43:1109-120. [10.1080/02522667.2022.2103302](https://doi.org/10.1080/02522667.2022.2103302)
18. Haghi KM, Madanipour M, Nikravan M, Asghari P, Mahdipour E: A systematic review of IoT in healthcare: applications, techniques, and trends. Journal of Network and Computer Applications. 2021, 192:103164. [10.1016/j.jnca.2021.103164](https://doi.org/10.1016/j.jnca.2021.103164)
19. Fira CM, Goras L: An ECG signals compression method and its validation using NNs. IEEE Transactions on Biomedical Engineering. 2008, 55:1319-26. [10.1109/tbme.2008.918465](https://doi.org/10.1109/tbme.2008.918465)
20. Hsieh JH, Hung KC, Liu JH, Wu TC: Wavelet-based quality-constrained ECG data compression system without decoding process. IEEE Multimedia. 2020, 27:33-45. [10.1109/mmul.2020.2983690](https://doi.org/10.1109/mmul.2020.2983690)
21. Jha CK, Kolekar MH: Empirical mode decomposition and wavelet transform based ECG data compression scheme. IRBM. 2021, 42:65-72. [10.1016/j.irbm.2020.05.008](https://doi.org/10.1016/j.irbm.2020.05.008)
22. Ibaida A, Abuadbbba A, Chilamkurti N: Privacy-preserving compression model for efficient IoMT ECG sharing. Computer Communications. 2021, 166:1-8. [10.1016/j.comcom.2020.11.010](https://doi.org/10.1016/j.comcom.2020.11.010)
23. Mohebbian MR, Wahid KA: ECG compression using optimized B-spline. Multimedia Tools and Applications. 2023, 82:21071-1083. [10.1007/s11042-023-14610-y](https://doi.org/10.1007/s11042-023-14610-y)
24. Abhishek S, Veni S: Sparsity enhancing wavelets design for ECG and fetal ECG compression. Biomedical Signal Processing and Control. 2022, 71:103082. [10.1016/j.bspc.2021.103082](https://doi.org/10.1016/j.bspc.2021.103082)
25. Ibaida A, Abuadbbba S, Al-Shammari D, Khalil I: ECG compression technique using fast fractals in the Internet of medical things. Concurrency and Computation: Practice and Experience. 2023, 35:e7812. [10.1002/cpe.7812](https://doi.org/10.1002/cpe.7812)
26. Pandey A: ECG data compression using the formation of QRS-complex segment bank and integer DCT-based plateau region processing. Biomedical Signal Processing and Control. 2023, 85:104823. [10.1016/j.bspc.2023.104823](https://doi.org/10.1016/j.bspc.2023.104823)
27. Singhai P, Kumar A, Ateek A, Ansari IA, Singh GK, Lee HN: ECG signal compression based on optimization of wavelet parameters and threshold levels using evolutionary techniques. Circuits, Systems, and Signal Processing. 2023, 42:3509-537.
28. Kumar SS, Ramachandran P: Nested compressive sensing for ECG signals using novel hybrid two-level approach of DWT and DCT sensing and sparsification. Discover Applied Sciences. 2025, 7:18. [10.1007/s42452-024-06374-3](https://doi.org/10.1007/s42452-024-06374-3)
29. Reddy PV, Tallapragada VVS: FPGA-enabled lossless ECG signal compression system using an integer adaptive compressor. Analog Integrated Circuits and Signal Processing. 2024, 119:331-61. [10.1007/s10470-024-02269-w](https://doi.org/10.1007/s10470-024-02269-w)