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# Marketing-Induced Fluctuations and Baseline Shifts in Product Adoption: A Change-Point Diffusion Analysis

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## Abstract

A major chunk of historical innovation diffusion models resembles a smooth S-shaped curve. In real life, this rate of diffusion keeps fluctuating due to various internal and external factors. Although studies have been proposed to incorporate the impact of measurable factors like price, promotion, and advertising-related variables, the timing process related to innovation adoption still continues to exhibit dynamic behavior. Hence, we have developed a diffusion model that is able to capture the non-stationarity of the diffusion process with respect to the time points at which a behavioral change in the diffusion process is observed, studied as the changepoint concept. The model is applied to the datasets of breakthrough innovations such as television, refrigerators, and clothes-drying machines. Incorporating changepoints into the generalized diffusion modeling framework markedly enhanced model performance, reducing mean squared errors by 58-99% and improving $R^2$ values notably, thereby substantially increasing estimation accuracy.

**Categories:** Innovation Economics, Social systems (economies, governments, industry), Mathematical programming
**Keywords:** adoption rate, change point, fluctuations, generalized bass model, innovation diffusion modelling
**JEL Classifications:** O33, C22, O44

## Introduction

The diffusion of innovation is a gradual and multifaceted process influenced by numerous economic, technological, social, and marketing factors. It often takes several years, or even decades, for an innovation to achieve widespread adoption in the market. The successful penetration of a new product depends on a complex interplay between consumer awareness, pricing, promotional intensity, competitive landscape, and technological advancement. Empirically, cumulative adoption for many innovations follows the familiar S-shaped pattern captured by classical models such as the Gompertz (Gompertz, 1825) and Logistic (Verhulst, 1838) curves. Among these, the Bass Model (BM) (Bass, 1969) remains the most influential, as it explicitly decomposes adoption into two behavioral mechanisms: innovation, governed by parameter $p$, which reflects adoption driven by external information sources (e.g., advertising, media), and imitation, governed by parameter $q$, which captures social contagion or word-of-mouth effects. Together with the market potential $m$, these parameters generate the canonical S-shape and provide an interpretable link between market drivers and adoption behavior.

In real-world settings, however, adoption often diverges from this idealized trajectory. The rate at which a new product or technology spreads through a population can fluctuate considerably in response to various market forces. Broadly, these fluctuations can be categorized as temporary (marketing-induced) shifts and permanent (structural) shifts. Temporary shifts arise from short-term marketing interventions such as advertising campaigns, promotional discounts, or pricing adjustments. These actions directly modulate consumers' awareness, perceived utility, and social influence, thereby altering the adoption likelihood represented by the innovation and imitation parameters. Short bursts of advertising temporarily raise $p$ by increasing exposure to external communication; campaigns that enhance product visibility or stimulate interpersonal influence temporarily elevate $q$; and price reductions affect both immediate adoption propensity and the effective size of the reachable market. Once the marketing effort subsides, adoption generally returns to its underlying trend. In contrast, permanent or structural shifts occur when more fundamental and enduring changes take place such as technological innovations, market deregulation, competitive entries, or changes in consumer preferences. These events create persistent alterations in diffusion dynamics, redefining the baseline parameters $p$, $q$, or $m$ that govern the innovation-imitation mechanism. Thus, the distinction between temporary and permanent fluctuations is intrinsically tied to the behavioral foundations of the diffusion process.

Recognizing and distinguishing between these two forms of shifts are critical for accurate modeling and managerial decision-making. Temporary effects inform tactical marketing responses, while permanent

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structural changes call for strategic adaptations to evolving market realities. This conceptual distinction underpins the current study, which seeks to integrate both short-term and long-term shifts within a unified innovation diffusion framework.

Marketing-induced effects, while temporary, can significantly influence early-stage product performance and are essential for evaluating tactical marketing decisions (Horsky and Simon, 1983) (Jain and Rao, 1990). Consequently, several studies have extended the classical BM to incorporate such effects, allowing the coefficients of innovation and imitation to vary with marketing variables (Kalish et al., 1985), (Kalish et al., 1986), (Jedidi et al., 1989), and (Ismail et al., 2012).The Generalized Bass Model (GBM) (Bass et al., 1994) addresses part of this limitation by allowing p(t) and q(t) to vary with marketing-mix variables such as price and advertising, modeled through a separable function of time and decision variables, collectively referred to as "current marketing efforts." These variables, denoted as x(t), influence the innovation and imitation parameters, enabling the model to capture short-term variations in adoption due to marketing activity. The incorporation of decision variables x(t), in the BM has been explained in equations (1) and (2).

$$\frac{dN(t)}{dt} = \left(p(t) + q(t)\frac{N(t)}{m}\right)(m - N(t)) \tag{1}$$

$$\frac{dN(t)}{dt} = \left(p + q\frac{N(t)}{m}\right)(m - N(t)).x(t) \tag{2}$$

On solving the model with initial condition at $t = 0, N(t) = 0$, the cumulative sales function takes the form:

$$N(t) = m\left[\frac{1 - exp\{-(p+q)X(t)\}}{1 + \frac{q}{p}exp\{-(p+q)X(t)\}}\right] \tag{3}$$

This extension successfully captures temporary deviations caused by tactical marketing actions. However, the GBM does not accommodate permanent, discontinuous shifts in diffusion dynamics arising from structural change-such as those triggered by technological discontinuities, market deregulation, or changes in consumer preferences-which require a different modeling treatment. it does not account for abrupt and enduring structural changes that permanently alter diffusion dynamics. Such permanent shifts may result from technological discontinuities, competitive entries, or macroeconomic events that reset the underlying parameters of innovation and imitation. To capture these regime changes, researchers have developed change-point models, which identify statistically significant breaks in diffusion trajectories and estimate distinct parameter sets for pre- and post-shift phases (Dekimpe and Hanssens, 2018) (Pauwels et al., 2002). To capture both short-term and long-term shifts, this study proposes an extension of the GBM that incorporates change points: (1) To account for temporary deviations resulting from marketing mix variables such as price and advertising, and (2) to represent permanent changes in the diffusion process due to more fundamental shifts in market conditions.

By allowing for discontinuities or shifts in parameters at identified change points, the model more accurately reflects the complexity and variability seen in real-world product life cycles. This dual-changepoint GBM framework enhances the ability to differentiate between temporary marketing effects and enduring changes in consumer adoption behavior, offering actionable insights for marketers and strategic planners.

Recently, machine learning and data-driven approaches have been applied to innovation diffusion modeling. For example, Bayesian estimation methods have been used to improve parameter inference under limited data scenarios (Ramírez-Hassan and Montoya-Blandón, 2020). Theoretical studies have further highlighted the limitations of learning diffusion models in small-sample environments, emphasizing the importance of model robustness and generalizability (Baek et al., 2021). In the context of distributed technologies, hybrid frameworks that combine diffusion theory with time-series forecasting techniques have shown practical value in scenario-based adoption forecasting (Ma et al., 2025). Also, AI-driven techniques such as agent-based simulations, reinforcement learning (RL), and sentiment-informed diffusion analyses have been applied to innovation diffusion research. For instance, Vinyals and fellows combine agent-based modeling with deep reinforcement learning to optimize adoption policies for digital tools in agriculture, illustrating how AI can tailor diffusion strategies to complex, dynamic environments (Vinyals et al., 2023). Similarly, Raman and others analyze ChatGPT adoption among university students through a mixed-method framework that integrates diffusion theory with sentiment analysis (Raman et al., 2024). These works demonstrate the growing potential of ML and AI in modeling adoption behavior. While these methods improve predictive performance, they often lack interpretability and direct links to managerial variables. Our study complements this stream of research by proposing a statistically interpretable framework that explicitly incorporates both transient marketing effects and persistent structural changes using a change-point augmented GBM approach (Table *1*).

| Feature | Bass Model (BM) | Generalized Bass Model (GBM) | ML-Based Approaches | Proposed Model |
|---|---|---|---|---|
| Core Variables | p, q, m | p(t), q(t), m | Data-driven (features vary) | p(t), q(t), m, change point τ |
| Marketing Factors Included | No | Yes (time-varying p, q) | Yes (often implicit or embedded) | Yes (explicitly modeled as transient effects) |
| Structural Change Handling | No | No | Yes (e.g., change-point detection, recurrent models) | Yes (explicit change-point representation) |
| Model Interpretability | High | High | Low to Moderate | High |
| Adaptability to Market Shifts | Low (static parameters) | Limited | Strong (adaptive models) | Strong (regime shift detection) |
| Managerial Insight | Basic | Good | Variable (depends on interpretability) | Strong (diagnostic and actionable) |
| Prediction Accuracy | Moderate | Improved over BM | High (with sufficient data) | High (with interpretable structure) |

**TABLE 1: Comparative summary of diffusion models**

Source: Author.

A change point refers to a specific moment in time at which the statistical properties or parameters of a process change-whether abruptly or gradually. In the context of diffusion modeling, a change point can signify a deterministic shift in the adoption rate, marking a permanent alteration in the underlying diffusion pattern. Researchers have argued that the diffusion process is often non-stationary, and thus a single, static S-shaped function cannot fully capture the evolving nature of adoption over an innovation's life cycle. Instead, diffusion should be viewed as a multiple-event timing process, with several distinct points at which key parameters change, reflecting the dynamic and multifaceted nature of real-world markets. It has been seen that there exists a discontinuity in the diffusion process between early and main market (Moore, 1991) as the adopters between the two markets are different in terms of their inclination and reluctance toward the innovation. Due to customer heterogeneity and limited communication between the two groups, a sales slump can be seen in early phases of Product Life Cycle (Goldenberg et al., 2002). Also, during the diffusion of innovation, there comes various interventions in the pathway for e.g., digital camera, automobile, etc. Due to technological advancement and other supportive strategic interventions, the performance of these innovations rose and hence, different rates of adoption can be experienced during different stage of life cycle (Guseo et al., 2007) (Yan et al., 2012). These changes can occur due to various exogenous (Political, Economic, Social, Technological, Legal, and Environmental factors i.e., PESTLE) and endogenous (price, income, advertising and customer's expectation, etc.) factors. The change point causes a structural break in the model, due to which parameters and functions of the model, like the diffusion or adoption rate, and hazard rate function, change. The fundamental model of this concept in the field of marketing is given by Kapur et al. (Kapur et al., 2007). The addition of the change point provides more accuracy to the diffusion models. This concept is highly used to study the fluctuations in adoption rate in case of dynamic market potential, multi-stage adoption, discrete logistic function, and consumer behavior-based innovation diffusion modeling (Anand et al., 2015), (Anand et al., 2018), and (Singh et al., 2012). Further, in others fields like software reliability engineering and social media analytics, similar concepts of structural change and adaptive modeling have been widely adopted (Chang, 1997) (Irshad et al., 2020). For instance, models increasingly account for human dynamics-such as learning, fatigue, and efficiency-when estimating fault detection and correction rates (Chang, 1997), (Singh et al., 2010), and (Samal and Kumar, 2024). Recent studies have emphasized how fault removal efficiency and human performance variability act as change-inducing factors in reliability growth models (Samal and Kumar, 2024) (Samal, 2025). These approaches demonstrate that accounting for temporal discontinuities and behavioral variability significantly enhances model accuracy and predictive performance.

Building on this foundation, the current study develops an extended GBM that incorporates change points to account for both transient and structural fluctuations in the diffusion process. Specifically, it distinguishes between short-term variations due to external marketing actions and long-term shifts in baseline diffusion dynamics resulting from underlying changes in the market environment or consumer behavior. The remainder of this paper is structured as follows: the research method is presented first, including model formulation and change-point integration. The model is then applied to real-world datasets, detailing parameter estimation and performance validation. This is followed by a discussion of the key findings and their managerial implications, emphasizing how the proposed framework can inform strategic marketing decisions. Finally, the study concludes with directions for future research.

# Research Method

## Notations

The notations and parameter definitions employed in the subsequent model formulation are presented in Table 2.

| ***N*(t)** | **Total number of adopters by time *t*** |
|---|---|
| *m* | Potential market size |
| □ | Change point |
| *p* | Rate at which product is adopted through external influence |
| *q* | Rate at which product is adopted under full internal influence |
| $b_i$, □$_i$ | Adoption rate and learning parameters before and after change point for *i* = 1, 2 respectively |
| *x*(t) | Current impact of marketing variables at time *t* |

**TABLE 2: Set of notations**

Source: Author.

## Model development

The GBM, though effective in incorporating marketing variables, is not fully suitable for modeling diffusion behavior in highly volatile or structurally shifting market environments. To address this limitation, an alternative formulation of the BM is adopted, in which the diffusion rate *b*(t) is treated as time-dependent and follows a logistic functional form, as proposed by Kapur et al. (Kapur et al., 2004). This dynamic specification allows the model to flexibly capture acceleration and deceleration patterns in adoption over time, reflecting both market learning and saturation effects.

The general diffusion process is represented as:

$$\frac{dN(t)}{dt} = b(t)\left(m - N\left(t\right)\right) \tag{4}$$

where:

$$b\left(t\right) = \frac{b}{1+\beta e^{-bt}} \tag{5}$$

Solving Equation (4) with initial condition: $t = 0, N(t) = 0$, we obtain:

$$N\left(t\right) = m\left[\frac{1-e^{-bt}}{1+\beta e^{-bt}}\right] \tag{6}$$

Where: b = p + q and $\beta = q \div p$

This expression (6) recovers the Bass diffusion model under the specified parameterization.

To capture short-term fluctuations driven by marketing activities, a marketing effort function, *x*(*t*), is introduced. It modifies the effective diffusion rate as follows:

$$\frac{dN(t)}{dt} = \left(\frac{b}{1+\beta e^{-bt}}\right)\left(m - N\left(t\right)\right)x(t) \tag{7}$$

The marketing effort function is given as: $x\left(t\right)=1+\gamma_1\frac{dPr(t)/dt}{Pr(t)}+\gamma_2\frac{dAdv(t)/dt}{Adv(t)}$

where:

$Pr(t)$ = price level at time *t*,

$Adv(t)$ = advertising expenditure at time *t*,

$\gamma_1$ and $\gamma_2$ = sensitivity coefficients capturing the influence of price and advertising, respectively.

Thus, $\gamma_1<0$ implies that a decrease in price increases adoption, while $\gamma_2>0$ reflects that higher advertising intensity promotes adoption.

Solving equation (7) again with initial condition: $t=0, N(t)=0$, we obtain:

$$N\left(t\right)=m\left[\frac{1-e^{-bX(t)}}{1+\beta e^{-bX(t)}}\right] \qquad (8)$$

Where: $X\left(t\right)=t+\gamma_1 Ln\frac{Pr(t)}{Pr(0)}+\gamma_2 Ln\frac{Adv(t)}{Adv(0)}$

Equation (8) provides an alternative formulation to the GBM by integrating time-varying marketing influences and dynamic diffusion behavior within a unified framework.

## Incorporating the structural break in diffusion process

Let $\tau$ denote a change point in the life cycle of a diffusion process for a technological product. The occurrence of this turning point leads to a shift in the diffusion rate, which may result in either an increase or a decrease in the rate, as given by equation (9):

$$b(t)=\begin{cases}\frac{b_1}{1+\beta_1 e^{-b_1 X(t)}}, & t\le\tau,\\ \frac{b_2}{1+\beta_2 e^{-b_2 X(t)}}, & t>\tau,\end{cases} \qquad (9)$$

Here, X(t) is an input function representing managerial control variables (e.g., advertising, pricing). While the model allows for a structural change in b(t) at time $\tau$, we do not impose any specific continuity or discontinuity on X(t) at $\tau$. Since X(t) is externally controlled, it may change smoothly or discretely depending on managerial decisions. The corresponding differential equation is then expressed as follows:

$$\frac{dN\left(t\right)}{dt}=\begin{cases}\frac{b_1}{1+\beta_1 e^{-b_1 X(t)}}\left[m-N(t)\right], & t\le\tau,\\ \frac{b_2}{1+\beta_2 e^{-b_2 X(t)}}\left[m-N(t)\right], & t>\tau,\end{cases} \qquad (10)$$

Solving the above equation with initial conditions, $t=0; N\left(t\right)=0$ and $t=\tau; N\left(t\right)=N\left(\tau\right)$, we get:

$$N(t)=\begin{cases}m\left[\frac{(1-e^{-b_1X(t)})}{(1+\beta_1e^{-b_1X(t)})}\right], & t\le\tau,\\ m\left[1-\left\{\frac{1+\beta_1}{1+\beta_1e^{-b_1X(\tau)}}\right\}\left\{\frac{1+\beta_2e^{-b_2X(\tau)}}{1+\beta_2e^{-b_2X(t)}}\right\}.e^{-\{b_1X(\tau)+b_2(X(t)-X(\tau))\}}\right], & t>\tau,\end{cases} \qquad (11)$$

Where: $X\left(t\right)=t+\gamma_1 Ln\frac{Pr(t)}{Pr(0)}+\gamma_2 Ln\frac{Adv(t)}{Adv(0)}$ and $b_i=p_i+q_i,\quad \beta_i=q_i/p_i$

The above function gives the total count of adopters by time *t*, under the change point framework. The inclusion of a change point allows the model to detect and quantify discrete shifts in adoption behavior. A positive shift in the innovation coefficient ($p_2>p_1$) may indicate the impact of stronger external influences, such as intensified advertising or a favorable market event. Conversely, a decrease in the imitation coefficient ($q_2<q_1$) may suggest diminished peer effects or market saturation after early adopters have influenced their networks. Further, next section deals with the real-life application of the proposed framework.

# Results And Discussion

## Parameter estimation

To check the effectiveness of inducing the change point methodology in the pioneered GBM, we have used the datasets (DS) from Bass et al. (Bass et al., 1994) pertaining to room air conditioners (AC), televisions (TV), and clothes dryers. These datasets are referred to as DS-1, DS-2, and DS-3, respectively. In this study, change point is estimated endogenously from the diffusion function rather than detected through external algorithms. This approach assumes that any structural shift in adoption should manifest as a change in the diffusion parameters $(p, q, m)$, making it behaviorally meaningful rather than a purely statistical disturbance. Because the data series is short and industry context indicates only one major intervention during the observation window, we restrict the model to a single change-point to avoid overfitting and ensure parameter identifiability. This choice represents a deliberate balance between model flexibility and robustness. Unlike generic change-point detection methods (e.g., CUSUM, PELT, Binary Segmentation), which detect distributional shifts independent of the underlying process, the diffusion-embedded change-point framework aligns the estimated break with actual changes in adoption dynamics. The parameter estimation of the proposed model is done on the described datasets. The parameters are determined through Statistical Package for the Social Sciences software using Non-linear Least Square (NLS) method. NLS was chosen because it directly minimizes the residual sum of squares between observed and predicted cumulative adoptions, without requiring restrictive assumptions about the distribution of errors. Compared to alternative approaches such as Ordinary Least Squares (OLS), Algebraic Estimates, or Maximum Likelihood Estimation (MLE), the NLS method provides more stable and accurate estimates for diffusion models, which are inherently nonlinear in their parameters (Bass, 1969) (Mahajan and Peterson, 1985). OLS and algebraic formulations often yield biased results due to their linearity assumptions, while MLE's efficiency depends on the correct specification of the underlying error distribution-a condition rarely satisfied in empirical adoption data. To ensure robustness, parameter estimation was repeated with multiple starting values and cross-checked against MLE-based optimization results. The estimated coefficients remained consistent across runs, and the relative model performance showed negligible variation, indicating that the NLS estimates are robust and reliable. The estimated parameters, standard errors, and 95% confidence intervals are summarized in Table *3*, Table *4*, and Table *5*.

**DS-1:** From Table *3*, the change-point for DS-1 is estimated at approximately time $\tau = 4.0$, indicating a distinct shift in the adoption trajectory. Following this point, the innovation coefficient ($p$) increases from 0.003 to 0.011, suggesting an intensified role of external influences such as media exposure or marketing activities in the post-change period. Conversely, the imitation coefficient ($q$) declines from 0.463 to 0.283, indicating a reduced dependence on interpersonal influence. The marketing-related parameters $\gamma_1$ (price effect) and $\gamma_2$ (advertising effect) exhibit theoretically consistent directions-negative for price and positive for advertising-though their confidence intervals include zero, reflecting statistical non-significance and weak marketing effects. This limitation implies price and advertisement changes may not meaningfully alter adoption rate; however, the directional signs remain behaviorally meaningful (Gelman and Stern, 2006). It indicates: (i) social diffusion mechanisms dominate over controllable marketing factors and (ii) maturity of the market-once awareness is widespread, external interventions matter less. However, in the context of short time series, confidence intervals may not always serve as a robust measure of model validity and should be interpreted with caution.

| Parameter | Estimate | Std. Error | 95% Confidence Interval | |
|---|---|---|---|---|
| | | | Lower Bound | Upper Bound |
| m | 20877.697 | 1278.859 | 17590.286 | 24165.109 |
| $p_1$ | 0.003 | 0.001 | 8.69E-05 | 0.006 |
| $q_1$ | 0.463 | 0.178 | 0.006 | 0.92 |
| $p_2$ | 0.011 | 0.005 | -3.00E-03 | 0.025 |
| $q_2$ | 0.283 | 0.035 | 0.192 | 0.374 |
| $\square$ | 4 | 0.708 | 2.179 | 5.821 |
| $\square_1$ | -0.825 | 0.693 | -2.607 | 0.957 |
| $\square_2$ | 0.291 | 0.339 | -0.58 | 1.162 |

## TABLE 3: Parameter estimates for DS-1

Source: Author.

**DS-2:** From Table 4, the change-point is estimated around time $\tau = 6.16$, signaling a mid-diffusion transition. Both pre- and post-change phases remain imitation-driven, suggesting that peer effects continued to dominate adoption decisions, albeit with slightly diminished influence. Also, strong negative effect of price and weak positive effect of advertising is captured through the data, but are not statistically significant, as indicated by confidence intervals encompassing zero. This reflect uncertainty of marketing effects on adoption rate possibly due to indirect effects, data sparsity, or strong parameter interdependence in nonlinear estimation.

| Parameter | Estimate | Std. Error | 95% Confidence Interval | |
|---|---|---|---|---|
| | | | Lower Bound | Upper Bound |
| m | 39265.722 | 823.848 | 35720.991 | 42810.453 |
| $p_1$ | 0.002 | 0 | 0 | 0.004 |
| $q_1$ | 0.716 | 0.026 | 0.605 | 0.827 |
| $p_2$ | 0.002 | 0.001 | -0.003 | 0.008 |
| $q_2$ | 0.654 | 0.095 | 0.244 | 1.063 |
| $\square$ | 6.157 | 0.489 | 4.055 | 8.26 |
| $\square_1$ | -2.983 | 0.883 | -6.781 | 0.814 |
| $\square_2$ | 0.106 | 0.142 | -0.504 | 0.715 |

## TABLE 4: Parameter estimates for DS-2

Source: Author.

**DS-3**: From Table 5, the model identifies a change-point at approximately time $\tau = 7.43$ for the cloth dryer dataset. The innovation coefficient increases from 0.006 to 0.012, while the imitation coefficient decreases from 0.354 to 0.233, indicating a gradual shift toward innovation-driven adoption. The advertising coefficient $(\gamma_1)$ is positive and statistically significant, reaffirming advertising's reinforcing role in stimulating awareness and imitation. The price effect $(\gamma_2)$ is positive but not statistically significant, possibly reflecting prestige or quality-signaling behavior among early adopters rather than a conventional price sensitivity pattern.

| Parameter | Estimate | Std. Error | 95% Confidence Interval | |
|---|---|---|---|---|
| | | | Lower Bound | Upper Bound |
| m | 20215.998 | 6745.908 | 2875.09 | 37556.906 |
| $p_1$ | 0.006 | 0.002 | 0.002 | 0.01 |
| $q_1$ | 0.354 | 0.038 | 0.256 | 0.452 |
| $p_2$ | 0.012 | 0.014 | -0.025 | 0.049 |
| $q_2$ | 0.233 | 0.181 | -0.233 | 0.698 |
| □ | 7.432 | 0.415 | 6.364 | 8.5 |
| □$_1$ | 0.429 | 0.821 | -1.681 | 2.54 |
| □$_2$ | 0.558 | 0.194 | 0.059 | 1.056 |

**TABLE 5: Parameter estimate of proposed model for DS-3**

Source: Author.

## Model validation

Model validation was performed to assess how effectively the proposed approach captures observed adoption dynamics relative to established diffusion models. Specifically, the performance of the Proposed Model was benchmarked against the Baseline BM and the GBM. The evaluation utilized two standard statistical indicators: the coefficient of determination ($R^2$), which represents the proportion of variance in the observed data that is explained by the model, and the mean squared error (MSE), which evaluates the model's predictive accuracy by measuring the average squared deviation between observed and predicted values. Higher $R^2$ and lower MSE values indicate superior model fit and forecasting reliability. Mathematically, they are expressed as:

$$R^2 = 1 - \frac{\sum_{i=0}^{n}\left(N(t_i) - \hat{N}(t_i)\right)^2}{\sum_{i=0}^{n}\left(N(t_i) - \bar{N}\right)^2}, \quad MSE = \frac{\sum_{i=0}^{n}\left(N(t_i) - \hat{N}(t_i)\right)^2}{n}$$

**DS-1:** Figure *1* illustrates the noncumulative adoption patterns predicted by BM, GBM, and proposed model. From the figure, it is evident that the BM captures the overall baseline trend of the diffusion process. The GBM, by incorporating external factors such as marketing, better captures short-term deviations and shifts in adoption rates. However, after time point 4, the gap between the GBM's predictions and actual data begins to widen. This divergence is accurately addressed by the proposed model, which estimates a structural break at approximately time 4.0 and incorporates the corresponding change in adoption dynamics. Table *6* summarizes the comparative model performance. The proposed model achieves the lowest MSE, with a 58.74% improvement over the GBM. It also demonstrates the highest $R^2$ value, confirming its superior fit.

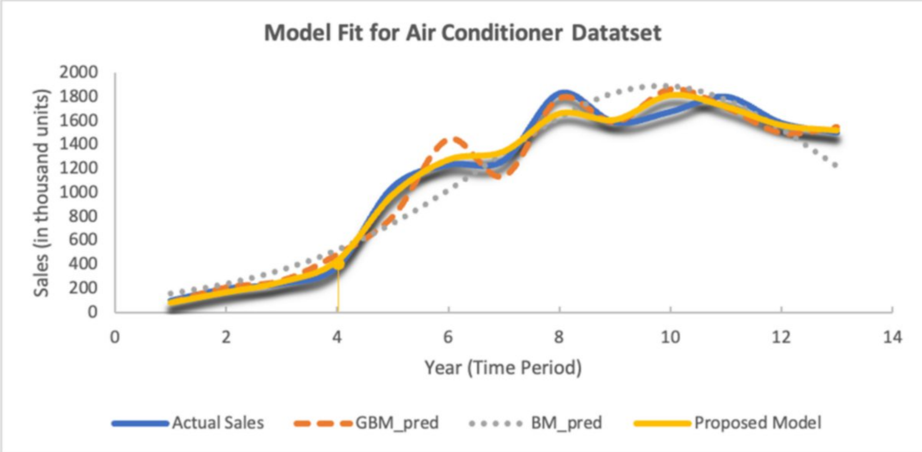


**FIGURE 1: Comparison of actual and predicted sales for DS-1**

Source: Author.

| Models | $R^2$ | MSE |
|---|---|---|
| BM | 0.99883253 | 31622.1245 |
| GBM | 0.99977734 | 5380.18785 |
| Proposed Model | 0.99990893 | 2219.7341 |

**TABLE 6: Performance of the proposed model in comparison with traditional models on DS-1**

Source: Author. MSE, mean squared error; GBM, Generalized Bass Model; BM, Bass Model

**DS-2**: Figure 2 presents the predicted noncumulative adoption patterns for the BM, GBM, and the proposed model, applied to the television dataset (DS-2). As observed, the BM struggles to capture deviations in adoption behavior, around time point 6, where the gap between predicted and actual values becomes more pronounced. The proposed model successfully accommodates this shift through the estimated change point and time-varying adoption dynamics, resulting in a significantly improved fit. As shown in Table 7, the proposed model achieves a 99.60% reduction in MSE compared to the GBM, along with a marginally higher $R^2$, highlighting its superior predictive performance.

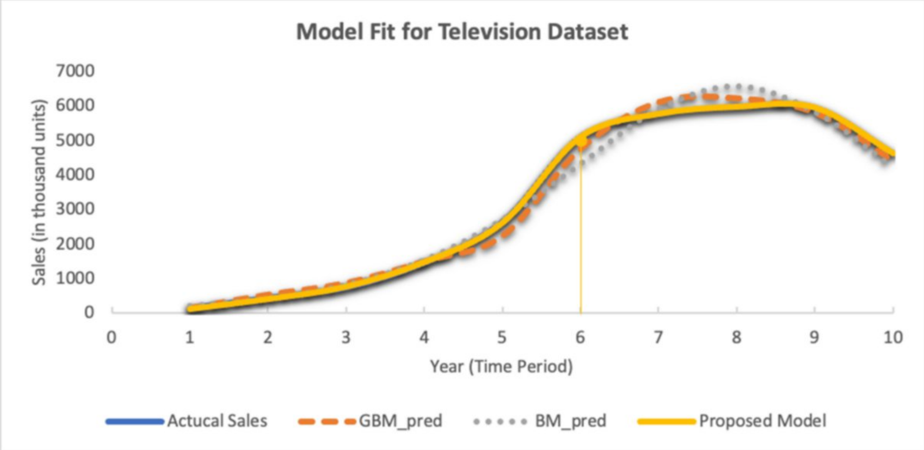


**FIGURE 2: Comparison of actual and predicted sales for DS-2**

Source: Author.

| Models | $R^2$ | MSE |
|---|---|---|
| BM | 0.99953771 | 50971.0833 |
| GBM | 0.99970909 | 33921.6034 |
| Proposed Model | 0.9999989 | 178.23809 |

**TABLE 7: Performance of the proposed model in comparison with traditional models on DS-2**

Source: Author. MSE, mean squared error; GBM, Generalized Bass Model; BM, Bass Model

**DS-3:** Figure 3 illustrates the predicted noncumulative adoption patterns for the BM, GBM, and the proposed model using the cloth dryer dataset (DS-3). As shown, the BM exhibits increasing deviation from the actual data, particularly in the interval [7,8]. This deviation is effectively captured by the proposed model through the incorporation of a structural change point and adjusted adoption parameters. As reflected in Table 8, the proposed model demonstrates a substantial improvement over GBM, achieving a 90.92% reduction in MSE. The corresponding increase in $R^2$ further supports the enhanced predictive accuracy of the model.

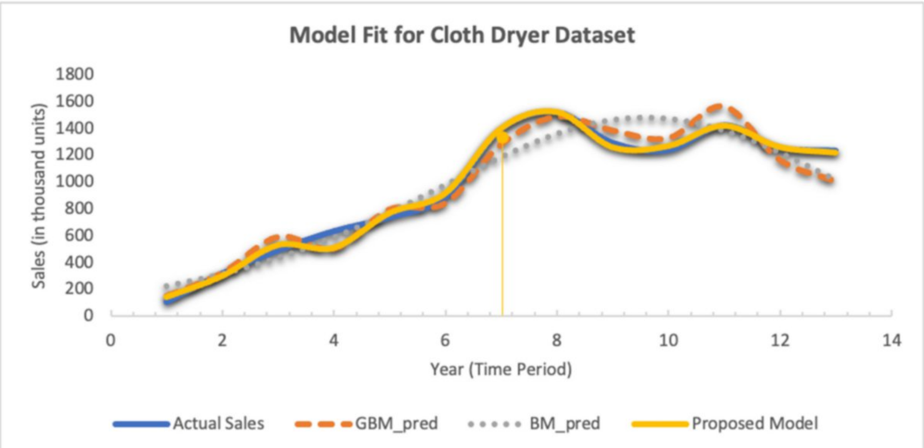


**FIGURE 3: Comparison of actual and predicted sales for DS-3**

Source: Author.

| Models | $R^2$ | MSE |
|---|---|---|
| BM | 0.99924279 | 14276.0362 |
| GBM | 0.99949329 | 9365.73946 |
| Proposed Model | 0.99995453 | 850.32783 |

**TABLE 8: Performance of the proposed model in comparison with traditional models on DS-3**

Source: Author. MSE, mean squared error; GBM, Generalized Bass Model; BM, Bass Model

## Discussion

This study employed an Innovation Diffusion Model (IDM) with change points to examine how market dynamics and marketing-mix effects evolve over time. The inclusion of change points allows the model to capture structural shifts in the diffusion process-such as those caused by new campaigns, competitor entry, or market saturation-rather than assuming constant parameters across the entire adoption curve. By incorporating the effects of price ($\gamma_1$) and advertising ($\gamma_2$), this framework extends the traditional BM to link marketing decisions with changes in innovation ($p$) and imitation ($q$) behavior.

Across datasets, market potential ($m$) estimates suggest moderately to highly penetrated markets, while the change points ($\tau$) identify distinct inflection periods in diffusion. Generally, innovation coefficients ($p$) increased slightly after the change points, whereas imitation coefficients ($q$) tended to decline-indicating that as markets mature, adoption becomes less dependent on peer influence and more responsive to external stimuli. This structural pattern aligns with classical diffusion theory.

The estimated effects of price ($\gamma_1$) and advertising ($\gamma_2$) were largely nonstatistically significant, except for a positive and significant advertising effect in DS-3. Rather than indicating that price and advertising have no impact, these results suggest that their influence is context-dependent and potentially indirect, often mediated through social diffusion mechanisms. Nonsignificant estimates also reflect statistical uncertainty due to limited sample size, model complexity, and possible multicollinearity among marketing variables. Thus, interpretations should focus on the direction and consistency of effects rather than their absolute significance. For instance, price consistently displayed a negative sign across most datasets-indicating likely price sensitivity-while advertising effects were consistently positive, supporting their role in stimulating late-stage adoption.

From a managerial perspective, this model equips decision-makers with actionable intelligence on how diffusion dynamics evolve and when interventions are most effective. However, rather than viewing these parameters abstractly, managers can translate specific parameter shifts into operational strategies:

(i) Interpreting the timing of structural shifts:
The estimated change point highlights when adoption patterns transition into a new regime (e.g., stronger innovation-driven or weaker imitation-driven growth). Even if no specific event is identifiable, this timing signals managers to re-evaluate the market's changed behavior.

(ii) Responding to parameter shifts:
Increases in $p$ (innovation effect) after a change point signal that external stimuli-like advertising or product updates-are gaining traction. Managers should intensify media spend or launch new campaigns during these windows to accelerate diffusion. Conversely, a decline in q (imitation effect) suggests waning peer influence or approaching market saturation, implying that word-of-mouth strategies may yield diminishing returns. At that stage, managers might focus on targeted retention or upgrade programs instead of broad awareness campaigns.

(iii) Acting on temporary marketing-induced shifts:
Short-term deviations in adoption caused by price and advertising-reflected through $\gamma_1$ and $\gamma_2$-help managers understand the immediate elasticity of the market. When adoption reacts sharply to promotional efforts (i.e., $|\gamma_1|$ or $\gamma_2$ relatively high), temporary interventions such as discounts, sales promotions, or increased media spend can be timed to coincide with periods where consumers are highly responsive. Alternatively, when these coefficients are weak, firms may avoid expensive short-term promotions and instead allocate resources toward long-horizon strategies.

(iv) Understanding marketing elasticity:
Even when $\gamma_1$ and $\gamma_2$ are not statistically significant, their estimated direction provides practical guidance. Negative $\gamma_1$ values indicate that lowering prices could support adoption during sensitive phases, while

positive $\gamma_2$ values, especially when significant as in DS-3, underscore the value of sustained advertising in later market stages when organic word-of-mouth slows.

Despite its strengths, the approach has limitations. Multiple change-point estimation increases model complexity and sensitivity to noise. The datasets analyzed, though representative, span limited adoption periods, constraining generalizability. Future extensions could integrate multi-product diffusion models or machine-learning-based regime detection to capture inter-market dependencies and improve predictive accuracy.

# Conclusions

Overall, the proposed framework bridges theoretical and practical gaps in innovation diffusion modeling. It provides a more adaptive representation of adoption dynamics while equipping managers with tools to detect, interpret, and respond to structural changes in real time. By linking parameter shifts directly to actionable strategies-such as reallocating marketing resources, adjusting prices, or timing product launches-firms can improve their strategic agility and sustain competitive advantage in rapidly changing markets.

However, the conclusions drawn here should be interpreted with caution. The datasets are relatively small, and several parameter estimates, particularly for price and advertising effects, were not statistically significant. This lack of significance reflects both statistical uncertainty and the inherent variability of real-world diffusion processes. Moreover, the assumption of discrete change points may oversimplify gradual behavioral shifts in the market. Future research should explore larger datasets, continuous-time change models, or Bayesian updating frameworks to improve robustness and real-time interpretability.

Despite these limitations, the IDM with change points remains a valuable framework for linking diffusion theory with practical marketing strategy. It enables managers not only to understand how adoption unfolds but also to recognize when and why the rules of diffusion change-transforming uncertainty into informed, data-driven action.

# Additional Information

## Author Contributions

All authors have reviewed the final version to be published and agreed to be accountable for all aspects of the work.

**Concept and design:** Khushboo Garg, Mohammed Shahid Irshad, Ompal Singh

**Acquisition, analysis, or interpretation of data:** Khushboo Garg

**Drafting of the manuscript:** Khushboo Garg

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## Disclosures

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The data analyzed in this work were sourced from Bass et al. (1994), with the full dataset provided in Table 7 of the published manuscript and are publicly available at https://www.jstor.org/stable/183674 .The processed datasets used in our analysis can also be provided by the corresponding author upon reasonable request.

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