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Impact of Technological Efforts for Conservation and Growth of Forestry Biomass: A Modeling Study

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Deepika Marwar [1] , Alok Malviya [1] , Shyam Sundar [2] , Maninder Singh Arora [3]

1. Department of Mathematics, Vikramajit Singh Sanatan Dharma (VSSD) College, Kanpur, IND 2. Department of Mathematics, Pranveer Singh Institute of Technology (PSIT), Kanpur, IND 3. Department of Mathematics, Pandit Prithi Nath College (PPN College), Kanpur, IND

Corresponding author: Deepika Marwar, deepika91marwar@gmail.com

Abstract

A nonlinear mathematical model is developed to analyze the impact of technological interventions and conservation efforts on forestry biomass. The study introduces a three-dimensional biomass-technology-effort model that uniquely integrates technological interventions with conservation efforts. Local and global stability are established through Lyapunov functions, and quantitative benchmarks (equilibrium levels, parameter sensitivities) are derived. The model represents an autonomous system of non-linear ordinary differential equations consisting of relationships among dynamic variables. These variables are the density of forestry biomass, the magnitude of technology and the magnitude of effort like reforestation. The analysis showed that augmenting technology and reforestation efforts can measurably enhance long-term biomass. The explicit numerical outcomes: equilibrium biomass ≈139 tons/ha (87% of L=160), technology ≈168 (thousand USD), effort ≈671 (million USD). Doubling \(\lambda\) increases biomass by ~0.06 units and technology by ~167 units. Doubling s_2 yields the largest biomass rise (~2.8 units in 50 years), while doubling \(\theta\) increases effort \(F\) by ~15-16 units. Sensitivity analysis confirms that biomass responds most strongly to s_1 and s_2, with \(\lambda\) and \(\theta\) mainly driving technology and effort. Lyapunov margins permit safe increase up to ≈7 times for \(\lambda\), \(\theta\), and s_2. Model trajectories initiating from different points in the positive region converge to the nontrivial equilibrium, demonstrating its stability. The study provides concrete numerical benchmarks for policymakers: even modest increases in technology-driven conservation can yield measurable gains in biomass growth and improved resilience against anthropogenic pressures.

Categories: Environmental and Sustainable Engineering, Environmental Engineering, Environmental Engineering and Sustainability
Keywords: forestry biomass, technological interventions, conservation effort, nonlinear model, sensitivity analysis, stability analysis, lyapunov method

Introduction

The world's forests are primarily classified into three categories: tropical, temperate, and boreal forests, which collectively account for approximately 85% of the world's biomass. About 4.06 billion hectares (ha), equivalent to 31% of the world's total land area, is covered by forests. Since 1990, there has been a loss of 178 million ha of forest, with a net decrease rate of approximately 4.7 million ha per year over the last decade. It is estimated that around 420 million ha of forest has disappeared globally due to deforestation since 1990 [1]. Although government has taken various steps to minimize deforestation, but we are a long away. Forest restoration and conservation using technology have been highly ignored [2,3]. That's why we are not able to get the required results, so it becomes necessary to adopt technology to achieve sustainable forest management goals.

Sustainable forest management is needed because forests serve as habitats for global terrestrial biodiversity and fulfill essential requirements for nearby human settlements. They are the backbone of the environment. They play a critical role in maintaining the ecological balance by mitigating global warming, preserving soil's water-holding capacity, controlling soil erosion, raising the underground water level, and serving as the largest carbon storehouse, essential for removing CO_2 from the environment [4,5]. Forests yield two types of non-wood forest products: plant-based and animal-based products, including medicines, aromatic products, fodder, food, meat, beeswax, honey, and more. Additionally, these forests provide employment opportunities for individuals living in villages near the forests, alleviating poverty.

The world’s population continues to grow unchecked and due to this the need for adequate food and shelter is also increasing. This has led to colonization, industrialization, tree cutting, and land utilization for agricultural production [6]. Multiple factors contribute to forest degradation, including human activities, fires, tree diseases, insects, legal-illegal logging, and severe climatic events [7-9]. Industries relying on wood-based and synthetic-based resources are also responsible for depleting forestry biomass. The migration of industries dependent on forest resources further exacerbates the depletion of forestry biomass within forest habitats [10].

Forest degradation results in a reduction in forest biodiversity values, productivity, and overall health. The diminishing forest area leads to the migration and extinction of species reliant on these forests [11]. Resource-dependent competing species decrease as population pressure intensify due to industrialization. Consequently, as population pressure increases, there is a corresponding decrease in forestry biomass, which adversely impacts the environment and downstream water quality [12].

Both global warming and forest biomass growth rates are inversely related. As global warming increases, there is a reduction in forestry biomass, while an increase in forest area leads to a decrease in global warming [13,14]. To address the adverse impacts of deforestation, the preservation of forests becomes imperative. The conservation of forest resources can be achieved through various means, including control over human activities, curbing toxicant practices, and preventing man-made fires [15]. Furthermore, the implementation of economic incentives, such as promoting the use of biofuels, and technological advancements is essential [3,16]. These efforts encompass activities like planting genetically engineered trees and educating people to explore alternatives to wood-based products [17-19]. For instance, in rural areas where wood is commonly used for cooking, providing efficient stoves and subsidies on such products can be highly effective [2]. Strictly prohibiting the extraction of foliage and bark, which constitute the primary carriers of micronutrients and sources of renewable energy production, is also crucial [20].

Maintaining forestry biomass requires timely and effective technological interventions, as prolonged delays can disrupt the ecosystem [21,22]. In this paper, we place particular emphasis on increasing forestry biomass through technology. Policymakers can adopt various technologies to enhance forestry biomass. These include utilizing drone photography and remote sensors to detect deforestation-affected areas. Geographic Information Systems (GIS), green-bots, and satellite imaging can be employed to assess forest health and monitor growth. Illegal logging can be identified and controlled using artificial intelligence, and blockchain technology can be used for transparency in reforestation and conservation efforts [23-25]. Blockchain ensures that donations are tracked and that funds for reforestation are used efficiently. As funds are deployed productively, efforts for reforestation and seedling initiatives increase. Green-bots are smart sensors attached to the bark of the trees in forests which send signals to the cloud, and with the help of internet we can collect information about any disturbance in the forests immediately, which helps us to take immediate and prompt action in time. Ultimately, the goal of increasing forestry biomass can be achieved.

In recent decades, several nonlinear mathematical models have been proposed to describe forest biomass dynamics under reforestation, afforestation, and economic interventions [16,19,22]. Earlier studies (e.g., [3,8,17]) examined the effects of population growth, industrialization, or generic conservation actions but rarely integrated technological factors explicitly. However, the growing role of digital monitoring, remote sensing, and automated restoration calls for models that capture technology-driven feedback in forest dynamics. The present study addresses this gap by developing a three-variable biomass-technology-effort model that couples technological innovation with ecological effort. The novelty of this model lies in integrating forestry biomass, technology, and conservation effort into a unified nonlinear framework, highlighting how modern technologies and human interventions can enhance forest growth through adaptive carrying capacity and feedback interactions, an approach rarely explored in traditional ecological models. This formulation provides new analytical results for both local and global stability using Lyapunov's method and quantifies how technological enhancement and reforestation jointly improve long-term biomass sustainability. As far as can be discerned, no mathematical model has been proposed using the concept of green bots or smart sensor technology to detect the health of and any disturbance in forests, which can be corrected in time by applying some efforts so that forestry biomass can grow. In the next section, a nonlinear mathematical model is formulated consisting of three variable combinations which are the density of forestry biomass, the magnitude of technology (like blockchain, green-bot and artificial intelligence), and the magnitude of effort (like reforestation, remedies for growth of forestry biomass density). In this study, it is shown that increase in such technologies, immediate information about any disturbance and problems like forest fires, insect attacks on trees, natural disasters, soil fertility, logging, overgrazing, and so on can be obtained. As information about the present condition is known with the help of technology, immediate efforts to rehabilitate the current situation, which are helpful for the growth of forestry biomass density, can be applied. In India, a few attempts have been made to increase the forestry biomass using such types of technology [25], and this study is helpful for the policymakers to determine the difference concerning existing methods, and it is much better for achieving the goal of sustainable forestry biomass growth.

Materials And Methods

Model formulation

This paper delves into the complex interplay between human activities, technology, reforestation efforts, and the growth of forestry biomass. It seeks to understand how the conservation of forests and reforestation initiatives with advanced technology can positively impact the growth of biomass, ultimately contributing to environmental stability and human well-being. To achieve this understanding, a three-dimensional nonlinear mathematical model is introduced.

The model presented here is an original formulation, designed to capture the dynamic interactions among forestry biomass B, technological advancement T, and conservation or reforestation effort F within a single nonlinear system of differential equations. The present model innovatively links these three components through feedback mechanisms: technology stimulates conservation effort, which in turn enhances both the growth rate and carrying capacity of the forest biomass. The adaptive carrying capacity term $L + s_2 F$ and the effort- and technology-driven interactions make this model distinctly different from other models. Formulation of model comprises of three state variables $B(t), T(t), F(t)$ in a geographical region, namely, the density of forestry biomass, the magnitude of technology (like blockchain technology, genetic engineering, drone photography, artificial intelligence, green bots technology, and more), and the magnitude of effort (like reforestation, conservation). The forestry biomass density is supposed to be governed by the logistic growth model [26]. The growth of biomass density depends upon efforts made to excel it. The logistic equation without effort F for B is as follows:

$$\frac{dB}{dt} = s(B - \frac{B^2}{L}) - s_0 B$$

Here, s is the intrinsic growth rate in forest biomass $B(t), L$ is the carrying capacity of $B(t)$ and s_0 represents the natural depletion coefficient for the loss in $B(t)$ due to insects and natural calamities.

To generalize, s_c and L_c are taken as function of F which is defined as follows:

$$s_c = s + s_1 F \text{ and } L_c = L + s_2 F$$

where, s_1 denotes the growth rate coefficient with respect to efforts for conservation and treatment against problems and s_2 represents the rate of contribution to carrying capacity of forest biomass due to effort (either seedling or planting trees). Now, the first equation of the model becomes:

$$\frac{dB}{dt} = s_c B - s\frac{B^2}{L_c} - s_0 B$$

In the early period of civilization, deforestation of forests was gradual and conditions were quite balanced. In the face of a growing global population and increased industrialization, it is imperative to find a balance between human needs and environmental preservation. Technological advancements play a pivotal role in this endeavor. Now, it is assumed that the current forestry biomass is $B(t)$ and its carrying capacity is L. We assume that technology is applied to the remaining biomass density $L - B(t)$, where L represents the carrying capacity of the forest. Hence, the growth rate of technology is directly proportional to $L - B(t)$ and it is assumed that the dynamics of technology governs logistic model [27]. Hence, the differential equation for growth rate of technology is as follows:

$$\frac{dT}{dt} = \lambda(L - B)T - \lambda_0 T^2$$

Here, $\lambda(L - B)$ is the intrinsic growth rate in technology and $\dfrac{\lambda(L - B)}{\lambda_0}$ is the carrying capacity of the technology.

Further it is assumed that efforts increase as technology increases, the growth rate of effort is directly proportional to technology applied [27]. The dynamics of the effort is given as follows:

$$\frac{dF}{dt} = \theta T - \theta_0 F$$

Here, θ denotes the growth rate coefficient of effort with respect to the technology. θ_0 represents natural depletion coefficient in implementation of efforts due to bad quality of seeds, less efficient labor work, and so on. The above equations can collectively construct the following system of nonlinear ordinary equations:

$$\left.\begin{aligned}\frac{dB}{dt} &= s_cB - s\frac{B^2}{L_c} - s_0B,\\ \frac{dT}{dt} &= \lambda(L-B)T - \lambda_0T^2,\\ \frac{dF}{dt} &= \theta T - \theta_0F\end{aligned}\right\} \quad (1)$$

where, $B(0)\geq 0, T(0)\geq 0, F(0)\geq 0.$ Initial conditions are taken to be positive and all the parameters are taken positive constants.

Positive bounds of variables

All variables of the model are positively bounded, it serves the condition for the well-proposed system (1). The bounds of the variables are represented by the following set:

Lemma 1. *The region of attraction which shows solutions of the dynamical system (1) are bounded, represented by the set* Ω:
$\Omega=\{(B,T,F)\in R^3_+ : 0\leq B\leq B_{max}, 0\leq T\leq T_{max}, 0\leq F\leq F_{max}\}$, *where,* $B_{max}=L_c(\frac{s_c-s_0}{s}), T_{max}=\frac{\lambda L}{\lambda_0}, F_{max}=\frac{\theta\lambda L}{\theta_0\lambda_0}$ *and this region attracts all the solution commencing inside the positive octant.*

Proof. Consider first equation of system (1) and using comparison principle, it is obtained

$$\frac{dB}{dt}\leq s_cB - s\frac{B^2}{L_c} - s_0B \implies \lim_{t\to\infty} supB(t)\leq L_c(\frac{s_c-s_0}{s}) = B_{max} \text{ (say)}$$

Also, applying comparison principle in equation second of the system (1), it is derived

$$\frac{dT}{dt}\leq \lambda(L-B)T-\lambda_0T^2 \implies \lim_{t\to\infty}\sup T(t)\leq \frac{\lambda L}{\lambda_0} = T_{\max} \text{ (say)}$$

Further, from third equation of the system (1), it is established

$$\frac{dF}{dt}\leq \theta T-\theta_0F \implies \lim_{t\to\infty} supF(t)\leq \frac{\theta\lambda L}{\theta_0\lambda_0} = F_{max} \text{ (say)}$$

This is the proof of lemma.

Analysis for equilibrium points

In this section, the feasible equilibrium states of the system is identified. The equilibrium solution of dynamical system can be obtained by equating whole system to zero. The model system has four feasible equilibria which are listed below:

$(i) H_0(0,0,0)$

$(ii) H_1(0, \frac{\lambda L}{\lambda_0}, \frac{\theta\lambda L}{\theta_0\lambda_0})$

$(iii) H_2(\frac{(s-s_0)L}{s}, 0, 0)$

$(iv) H^*(B^*, T^*, F^*)$

The existence of H_0: The existence of H_0 is obvious, if $B=0$ is taken from first equation of model (1) and $T=0$ from second equation of the system (1) then $F=0$ is obtained. Hence, the equilibrium point is $H_0(0,0,0)$

The existence of H_1: For the existence of equilibrium state H_1, $B=0$ from first equation of the system (1) is taken and $T\neq 0$. H_1 can be obtained solving below equations:

$$\lambda(L-B)-\lambda_0T=0 \quad (2)$$

$$\theta T-\theta_0F=0 \quad (3)$$

Here, $T=\frac{\lambda L}{\lambda_0}$ and $F=\frac{\theta\lambda L}{\theta_0\lambda_0}$ is obtained. Hence, the equilibrium point is $H_1(0,\frac{\lambda L}{\lambda_0},\frac{\theta\lambda L}{\theta_0\lambda_0})$

The existence of H_2: Equilibrium point H_2 exists, if $B\neq 0$ is taken from first equation of the model system (1), $T = 0$ from second equation of model (1) and solve the following algebraic expressions:

$$s_c - s\frac{B}{L_c} - s_0 = 0 \qquad (4)$$
$$\theta T - \theta_0 F = 0 \qquad (5)$$

$B = \dfrac{(s-s_0)L}{s}$ and $F = 0$ is obtained. Hence, the equilibrium point is

$$H_2(\frac{(s-s_0)L}{s}, 0, 0)$$

The existence of H^* : Take $B \neq 0$ and $T \neq 0$ from first and second equation the model (1) respectively and solve the following algebraic expressions:

$$s_c - s\frac{B}{L_c} - s_0 = 0 \qquad (6)$$
$$\lambda(L - B) - \lambda_0 T = 0 \qquad (7)$$
$$\theta T - \theta_0 F = 0 \qquad (8)$$

After solving above expression, a quadratic equation in F is obtained as follows:

$$s_1 s_2 F^2 + \left[s_2(s-s_0) + s_1 L + \frac{\theta_0\lambda_0 s}{\theta\lambda}\right] F - s_0 L = 0$$

Above quadratic equation has unique positive root F^* as follows:

$$F^* = \frac{-\left[s_2(s-s_0) + s_1 L + \frac{\theta_0\lambda_0 s}{\theta\lambda}\right] + \sqrt{\left[s_2(s-s_0) + s_1 L + \frac{\theta_0\lambda_0 s}{\lambda\theta}\right]^2 + 4 s_0 s_1 s_2 L}}{2 s_1 s_2}$$

Let F^* be the positive root then $T^* = \dfrac{\theta_0 F^*}{\theta}$ and $B^* = \dfrac{(s + s_1 F^* - s_0)(L + s_2 F^*)}{s}$.
Hence, $H^*(B^*, T^*, F^*)$ is the non-trivial equilibrium point.

Stability analysis

The stability of the system (1) is proved in this section. Here, the behavior of system in the local and global neighborhood is examined during this analysis.

Local stability

The system becomes locally stable if it settles to the equilibrium point with respect to small perturbation. In order to perceive the local stability of the system, first linearize it in the vicinity of the equilibrium point. After this, the variational matrices corresponding to the equilibrium points H_0, H_1, H_2 respectively is obtained. Then, the eigen values of these Jacobain matrices and the stability of equilibrium points can be determined according the sign of eigen values. The local stability of H^* is accomplished by applying Lyapunov's direct method. The concise result is provided in the following theorem:

Theorem 1. *The equilibria* H_0, H_1 *and* H_2 *are unstable. The equilibrium* H^* *is locally asymptotically stable if the inequality given below holds:*

$$\lambda^2\theta^2\{s_1(L + s_2F^*)^2 + s s_2 B^*\}^2 < \theta_0^2\lambda_0^2 s^2 (L + s_2 F^*)^2$$

Proof. The stability of equilibrium points H_0, H_1, H_2 of the model system (1) can be obtained by finding Jacobian matrices J_0, J_1 and J_2 at H_0, H_1 and H_2 respectively.
Now, J_0 corresponding to H_0 is as follows:
\(J_0 =\begin{pmatrix}
s-s_0&0&0\\
0&\lambda L&0\\
0&\theta&-\theta_0\\
\end{pmatrix}\)
Clearly from J_0, eigen values are $s - s_0$, λL and $-\theta_0$. Here, J_0 has two positive eigen values in $B - T$-direction. Hence, H_0 is unstable.
Again, Jacobian matrices J_1 and J_2 corresponding to H_1 & H_2 are as follows:
\(J_1 =\begin{pmatrix}
s-s_0+\dfrac{s_1\theta\lambda L}{\lambda_0\theta_0}&0&0\\
-\dfrac{\lambda^2 L}{\lambda_0}&-\lambda L&0\\

0&\theta&-\theta_0\\
\end{pmatrix}\)
& \(J_2 =\begin{pmatrix}
s-s_0&0&\dfrac{(s-s_0)[s_1L+s_2(s-s_0)]}{s}\\
0&\dfrac{s_0 \lambda L}{s}&0\\
0&\theta&-\theta_0\\
\end{pmatrix}\)

J_1 and J_2 have one positive eigen value. Hence, H_1 is unstable in B-direction and H_2 is unstable in $B-T$-direction.

Now, the Jacobian matrix J^* corresponding to H^* is as follows:
\(J^* =\begin{pmatrix}
-\dfrac{sB^*}{(L+s_2F^*)}&0&s_1B^*+\dfrac{ss_2B^{*2}}{(L+s_2F^*)^2}\\
-\lambda T^*&-\lambda_0 T^*&0\\
0&\theta&-\theta_0\\
\end{pmatrix}\)

The eigen value for this matrix J^* can't be easily evaluated by Routh-Hurwitz criteria. Hence, the criterion for the local stability of H^* can be determined considering some suitable Lyapunov function which is stated as follows:

First, linearize the system (1) about $H^*(B^*,T^*,F^*)$ by using the transformations
\(\begin{align}\tag{9}
B=B^*+b, T=T^*+t_1, F=F^*+f
\end{align}\)

where, b, t_1, f are small perturbation around H^*

Take a positive definite function about H^*,
\(\begin{equation}\tag{10}
V=\dfrac{1}{2}\dfrac{m_1b^2}{B^*}+\dfrac{1}{2}m_2\dfrac{t_1^2}{T^*}+\dfrac{1}{2}m_3f^2
\end{equation}\)

where, $m_1>0$, $m_2>0$, $m_3>0$ to be chosen properly.

Now, differentiating Equation (10) with respect to t to get
\(\begin{equation}\tag{11}
\dfrac{dV}{dt}=\dfrac{m_1b}{B^*}\dfrac{db}{dt}+\dfrac{m_2t_1}{T^*}\dfrac{dt_1}{dt}+m_3f\dfrac{df}{dt}
\end{equation} \)

Using the model equations in Equation (11) and after simplification it is derived
\(\begin{align}\tag{12}
\dfrac{dV}{dt}&=m_1(s_1+\dfrac{ss_2B^*}{(L+s_2F^*)^2})fb-m_1\dfrac{sb^2}{L+s_2F^*}-m_2(\lambda bt_1+\lambda_0t_1^2)+m_3(\theta ft_1-\theta_0f^2)
\end{align}\)

Now, the sufficient conditions for $\dfrac{dV}{dt}$ come out to be negative definite are given below:
\(\begin{gather}
m_1^2\{s_1+\dfrac{ss_2B^*}{(L+s_2F^*)^2}\}^2
<\dfrac{m_1m_3s\theta_0}{L+s_2F^*} \tag{13}
\end{gather}\)
\(\begin{gather}
\lambda^2m_2^2<\dfrac{m_1m_2s\lambda_0}{L+s_2F^*}\tag{14}
\end{gather}\)
\(\begin{gather}
\theta^2m_3^2<m_3m_2\theta_0\lambda_0
\tag{15}
\end{gather}\)

After choosing, $m_2=1$ and $m_1=1$ it is established
\(\begin{align} \tag{16}
\lambda^2\theta^2\{s_1(L+s_2F^*)^2+ss_2B^*\}^2<m_3<\theta_0^2\lambda_0^2s^2(L+s_2F^*)^2
\end{align}\)

This is the required condition. If the above condition (16) is satisfied then system is locally stable and the interior equilibrium H^* is locally stable.

Global stability

Here in this section, the convergence of solution trajectories with respect to H^* starting inside the region Ω of system (1) is discussed. The concise result is provided in the following theorem:

Theorem 2.

The equilibrium H^* is globally asymptotically stable in Ω if the inequality given below holds:

$$\lambda^2\theta^2\{s_1(L+s_2F^*)(L+s_2\frac{\theta\lambda L}{\theta_0\lambda_0})+ss_2B^*\}^2<\lambda_0^2\theta_0^2s^2\{L+s_2F^*\}^2$$

Proof. Take a positive definite function

$$\begin{align}\tag{17}
W&=k_1(B-B^*-B^*\ln\dfrac{B}{B^*})+k_2(T-T^*-T^*\ln\dfrac{T}{T^*})+\dfrac{k_3}{2}(F-F^*)^2
\end{align}$$

where, $k_1>0,\quad k_2>0,\quad k_3>0$ to be chosen properly.

Now, differentiating Equation (17) of function W with respect to t to get,

$$\begin{align}\tag{18}
\dot{W}&=k_1(1-\dfrac{B^*}{B})\dfrac{dB}{dt}+k_2(1-\dfrac{T^*}{T})\dfrac{dT}{dt}+k_3(F-F^*)\dfrac{dF}{dt}
\end{align}$$

Using model system (1) equations in Equation (18) to get,

$$\begin{align}\tag{19}
\dot{W}&=-k_1\{\dfrac{sL+ss_2F^*}{(L+s_2F^*)(L+s_2F)}\}(B-B^*)^2-k_2\lambda_0(T-T^*)^2-k_3\theta_0(F-F^*)^2\\
&+k_1\{s_1+\dfrac{ss_2B^*}{(L+s_2F^*)(L+s_2F)}\}(B-B^*)(F-F^*)-k_2\lambda(B-B^*)(T-T^*)\\
&+k_3\theta(F-F^*)(T-T^*)
\end{align}$$

$\dot{W}$ comes out to be negative definite if the following inequalities hold

$$\begin{gather}
k_1^2\{s_1+\dfrac{ss_2B^*}{(L+s_2F^*)(L+s_2F)}\}^2<k_1k_3\theta_0\dfrac{s(L+s_2F^*)}{(L+s_2F^*)(L+s_2F)}\tag{20}
\end{gather}$$

$$\begin{gather}
k_2^2\lambda^2<k_1k_2\lambda_0\dfrac{s(L+s_2F^*)}{(L+s_2F^*)(L+s_2F)}\tag{21}
\end{gather}$$

$$\begin{gather}
k_3^2\theta^2<k_2k_3\theta_0\lambda_0\tag{22}
\end{gather}$$

Solving Equations (20)-(22) and taking $k_1=1,\quad k_2=1$ inside the region of attraction Ω, it is derived

$$\begin{align}\tag{23}
\lambda^2\theta^2\{s_1(L+s_2F^*)(L+s_2F_{max})+ss_2B^*\}^2<k_3<\theta_0^2\lambda_0^2s^2(L+s_2F^*)^2
\end{align}$$

Now, putting $F_{max}=\dfrac{\theta\lambda L}{\theta_0\lambda_0}$ in above Equation (23), it is obtained

$$\begin{align}\tag{24}
\lambda^2\theta^2\{s_1(L+s_2F^*)(L+s_2\dfrac{\theta\lambda L}{\theta_0\lambda_0})+ss_2B^*\}^2<k_3<\theta_0^2\lambda_0^2s^2(L+s_2F^*)^2
\end{align}$$

If the inequality (24) holds then $\dfrac{dW}{dt}$ is negative definite. Hence, the interior equilibrium point H^* is globally asymptotically stable inside Ω. This is the complete proof the theorem.

Remark

The analysis of the local and global stability conditions reveals that the parameters s_1, s_2, λ, and θ tend to reduce the stability of the system's equilibrium points. An increase in the magnitude of these parameters may drive the system away from its stable state H^*.

Description of parameter	Symbol	Units and values
Intrinsic growth rate of forest biomass	s	0.6 per year
Carrying capacity of forest biomass	L	160 tons per hectare
Natural depletion in forest biomass	s_0	0.09 per year
Intrinsic growth rate of forest biomass due to effort	s_1	0.000001 per million USD per year
Rate of contribution to carrying capacity of forest biomass due to effort	s_2	0.005 tons per hectare per million USD
Growth rate coefficient of technology	λ	0.008 hectare per ton per year
Natural depletion in technology	λ_0	0.001 per thousand USD per year
Growth rate coefficient of effort	θ	0.002 million USD per thousand USD per year
Natural depletion in effort	θ_0	0.0005 per year
Time	t	year
Density of forestry biomass	B	tons per hectare
The magnitude of technology	T	Budget allocation for research and development (USD in thousands)
The magnitude of effort	F	Budget allocation for the effort implementation (USD in millions)

TABLE 1: Description table for various parameters and variables

Results And Discussion

Analytical results showed that the proposed model admits four non-negative equilibria, of which three are boundary equilibria and one is a unique nontrivial interior equilibrium. The three boundary equilibria were found to be unstable, whereas the nontrivial equilibrium is stable under certain conditions. The local and global stability of this equilibrium was established using Lyapunov's method. Now, the numerical simulations supported these findings is as follows:

Numerical simulation

Feasibility of the system is necessary for the corresponding real-world problems and it can be examined by conducting some numerical computation using MATLAB. To perform the numerical simulation, some values of parameters is taken from the previously available literature [2,10]. The values of these parameters provided in the Table 1 with their description are as follows:

$s = 0.6, \quad L = 160, \quad s_0 = 0.09, \quad s_1 = 0.000001,$
$s_2 = 0.005, \quad \lambda_0 = 0.001, \quad \lambda = 0.008,$
$[\theta = 0.002], \quad \theta_0 = 0.0005$. For the baseline parameter set (Table 1), the system converged to:

⊠ Biomass density B^*≈139.0 \approx 139.0 tons/ha (about 87% of the carrying capacity, L=160 tons/ha),
⊠ Technology level T^*≈168 \approx 168 (thousand USD), and
⊠ Effort level F^*≈671 \approx 671 (million USD).

In Figure 1, the variation of forestry biomass for different values of s_1, θ, λ, s_2 respectively is plotted. These plots show that increase in value of s_1, θ, λ, s_2 increase the forest biomass $B(t)$. It is noticed from Figure 2 that technology increases with increase in λ but as the values of parameters s_1, θ and s_2 are raised, there is a negative impact on the technology. In Figure 3, variation in effort is illustrated for different values of s_1, θ, λ and s_2 respectively. In Figure 3, it is seen that effort increases as the value of θ and λ increase. It is observed that with the increase of value of s_1 and s_2, effort decreases. This is expected outcome, since s_1 and s_2 are the growth rate coefficient for forestry biomass with respect to effort.

In Figure 4, the global stability plots of the system are shown. Two-dimensional global stability in the $T-F$-plane and $B-T$-plane is shown in the first two plots of Figure 4, and here it is noted that all the solution trajectories starting inside the region of attraction converge to the equilibrium point. Similarly, the third plot of Figure 4 depicts the convergence of all the solution trajectories to the equilibrium point commencing inside the region of attraction in $B-T-F$-space. This indicates that under sustained inputs, forest biomass stabilizes close to its ecological maximum while requiring substantial but finite technological and conservation investments.

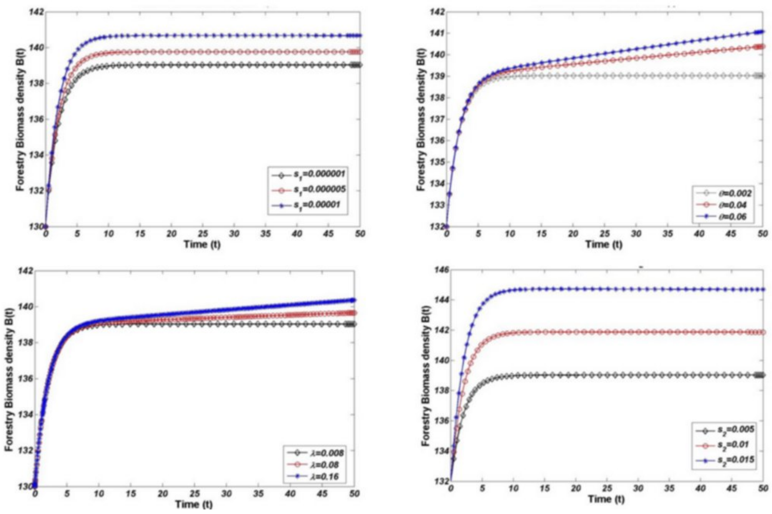

FIGURE 1: Variation in forestry biomass density B for distinct values of s_1, θ, λ, and s_2

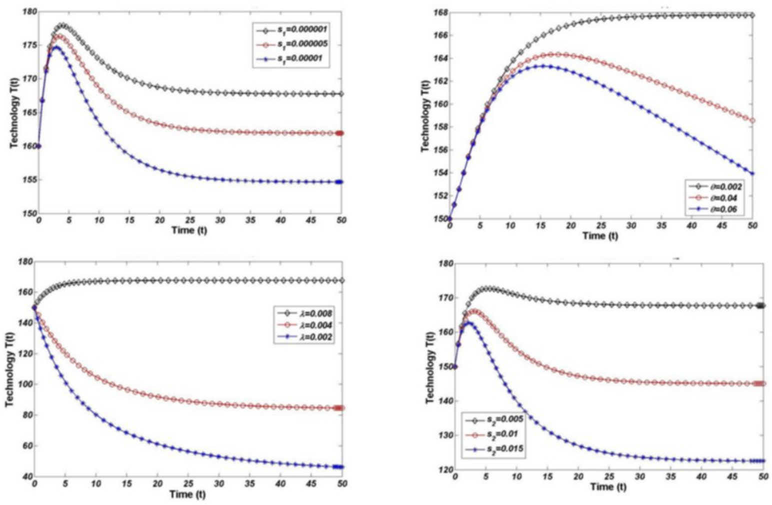

FIGURE 2: Variation in technology T for distinct values of s_1, θ, λ, and s_2

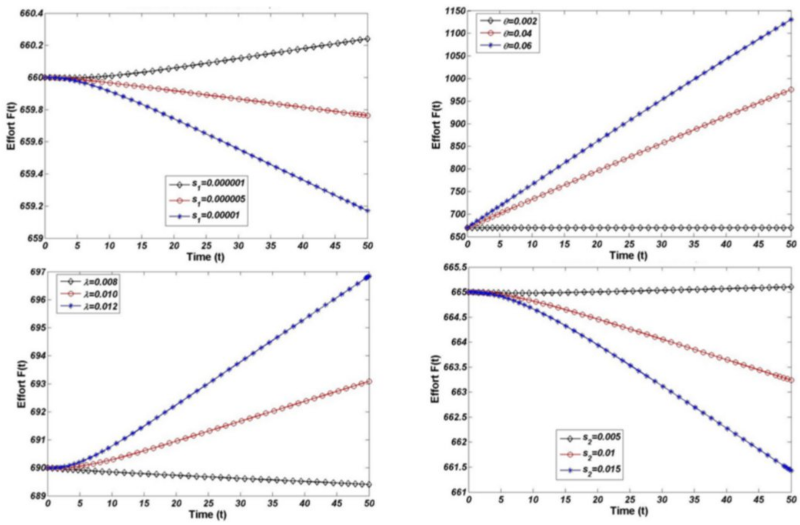

FIGURE 3: Variation in effort F for distinct values of s_1, θ, λ, and s_2

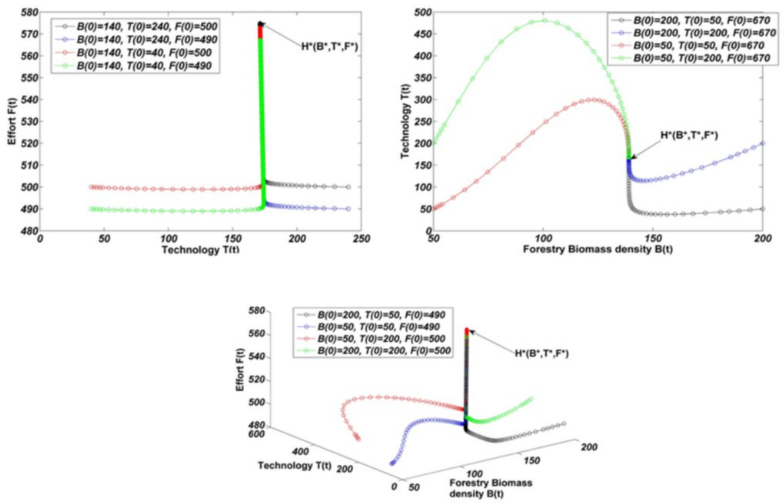

FIGURE 4: Global stability of T*-F*, B*-T*, and B*-T*-F* in T-F-plane, B-T-plane, and B-T-F-space, respectively

Sensitivity analysis

Here in this section, the interconnection between state variables and the parameters θ, λ, s_1 and s_2 of the model (1) is perceived by performing the semi-relative basic differential sensitivity analysis [28]. $q\dfrac{\partial Z(t,q)}{\partial q}$ represents the semi-relative solution for Z (variable) with respect to parameter q. These solutions furnish information regarding the effect of doubling parameters on the state variables.

Now, the semi-relative sensitivity system for state solutions with respect to parameter θ is as follows:

\(\begin{equation} \tag{25}
\left.
\begin{aligned}
\dot{B_\theta}(t,\theta)&=(s+s_1F(t,\theta))B_{\theta}(t,\theta)+s_1F_{\theta}(t,\theta)B(t,\theta)-\dfrac{2sB(t,\theta)B_{\theta}(t,\theta)}{(L+s_2F(t,\theta))}\\
&+\dfrac{ss_2B^2(t,\theta)F_{\theta}(t,\theta)}{(L+s_2F(t,\theta))^2}-s_0B_{\theta}(t,\theta),\\
\dot{T_\theta}(t,\theta)&=\lambda(L-B(t,\theta))T_{\theta}(t,\theta)-\lambda B_{\theta}(t,\theta)T(t,\theta)-2\lambda_0T(t,\theta)T_{\theta}(t,\theta),\\
\dot{F_\theta}(t,\theta)&= T(t,\theta)+\theta T_{\theta}(t,\theta)-\theta_0F_{\theta}(t,\theta)

\end{aligned}
\right\}
\end{equation}\)

Again, the semi-relative sensitivity system for state variables with respect to parameter λ is as follows:
\(\begin{equation}\tag{26}
\left.
\begin{aligned}
\dot{B_\lambda}(t,\lambda)&=(s+s_1F(t,\lambda))B_{\lambda}(t,\lambda)+s_1F_{\lambda}(t,\lambda)B(t,\lambda)-\dfrac{2sB(t,\lambda)B_{\lambda}(t,\lambda)}{(L+s_2F(t,\lambda))}\\
&+\dfrac{ss_2B^2(t,\lambda)F_{\lambda}(t,\lambda)}{(L+s_2F(t,\lambda))^2}-s_0B_{\lambda}(t,\lambda),\\
\dot{T_\lambda}(t,\lambda)&=(L-B(t,\lambda))T(t,\lambda)+\lambda(L-B(t,\lambda))T_{\lambda}(t,\lambda)-\lambda B_{\lambda}(t,\lambda)T(t,\lambda)\\
&-2\lambda_0T(t,\lambda)T_{\lambda}(t,\lambda),\\
\dot{F_\lambda}(t,\lambda)&= \theta T_{\lambda}(t,\lambda)-\theta_0F_{\lambda}(t,\lambda)
\end{aligned}
\right\}
\end{equation}\)

Again, the semi-relative sensitivity system with respect to parameter s_1 is as follows:
\(\begin{equation} \tag{27}
\left.
\begin{aligned}
\dot{B_{s_1}}(t,s_1)&=F(t,s_1)B(t,s_1)+(s+s_1F(t,s_1))B_{s_1}(t,s_1)+s_1F_{s_1}(t,s_1)B(t,s_1)\\
&-\dfrac{2sB(t,s_1)B_{s_1}(t,s_1)}{(L+s_2F(t,s_1))}+\dfrac{ss_2B^2(t,s_1)F_{s_1}(t,s_1)}{(L+s_2F(t,s_1))^2}-s_0B_{s_1}(t,s_1),\\
\dot{T_{s_1}}(t,s_1)&=\lambda(L-B(t,s_1))T_{s_1}(t,s_1)-\lambda B_{s_1}(t,s_1)T(t,s_1)-2\lambda_0T(t,s_1)T_{s_1}(t,s_1),\\
\dot{F_{s_1}}(t,s_1)&= \theta T_{s_1}(t,s_1)-\theta_0F_{s_1}(t,s_1)
\end{aligned}
\right\}
\end{equation}\)

Further, the semi-relative sensitivity system with respect to parameter s_2 is as follows:
\(\begin{equation}\tag{28}
\left.
\begin{aligned}
\dot{B_{s_2}}(t,s_2)&=(s+s_1F(t,s_2))B_{s_2}(t,s_2)+s_1F_{s_2}(t,s_2)B(t,s_2)-\dfrac{2sB(t,s_2)B_{s_2}(t,s_2)}{(L+s_2F(t,s_2))}
\\
&+\dfrac{sB^2(t,s_2)F(t,s_2)}{(L+s_2F(t,s_2))^2}+\dfrac{ss_2B^2(t,s_2)F_{s_2}(t,s_2)}{(L+s_2F(t,s_2))^2}-s_0B_{s_2}(t,s_2),\\
\dot{T_{s_2}}(t,s_2)&=\lambda(L-B(t,s_2))T_{s_2}(t,s_2)-\lambda B_{s_2}(t,s_2)T(t,s_2)-2\lambda_0T(t,s_2)T_{s_2}(t,s_2),\\
\dot{F_{s_2}}(t,s_2)&= \theta T_{s_2}(t,s_2)-\theta_0F_{s_2}(t,s_2)
\end{aligned}
\right\}
\end{equation}\)

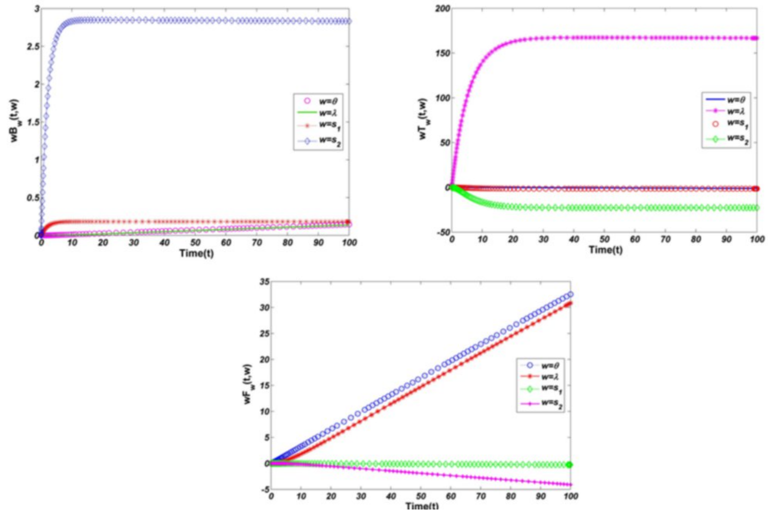

FIGURE 5: Plots of semi-relative basic differential senstivity taking parameters θ, λ, s_1, and s_2 for different state variables

In Figure 5, the plots of semi-relativity basic differential sensitivity are demonstrated for distinct state variables $B(t)$, $T(t)$ and $F(t)$ with respect to parameters θ, λ, s_1, s_2. Herein, in the first plot of Figure 5, all the parameters have a positive impact on B. It is observed that when the values of θ, λ, s_1, s_2 are doubled, forestry biomass increases by 0.06 tons/ha, 0.06 tons/ha, 0.18 tons/ha, and 2.8 tons/ha, respectively, in 50 years. From the second plot of Figure 5, it is noticed that only parameter λ has a positive effect and the remaining parameters have a negative effect on the technology applied. As the parameter λ is doubled, the value of technology increases by 167.2 thousand USD in 50 years. Again, from the third plot of the Figure 5, it is concluded that two parameters λ and θ are used to increase the effort by 15 and 16.3 million USD respectively in 50 years, showing that technological progress amplifies conservation actions on the ground. In contrast, larger values of s_1 and s_2 reduced the long-term equilibrium value of the effort variable F, because more productive ecosystems require proportionally less sustained human input. Sensitivity analysis confirmed that biomass is most responsive to s_2 (carrying-capacity augmentation) and s_1 (growth enhancement), while λ and θ primarily influence technology and effort with moderate indirect effects on biomass.

Discussion

The results of the present study align well with existing literature, showing that the integration of technological advancement and conservation effort can significantly enhance forest biomass growth and stability. Similar to previous studies based on the logistic growth concept [26,27], the model confirms that forest biomass follows density-dependent regulation, but its growth and carrying capacity can be improved through sustained conservation activities. The positive feedback between technology and conservation effort reflects findings in recent works highlighting the role of AI, drones, and blockchain technology in effective forest monitoring and management. Thus, the proposed model not only supports existing ecological and technological theories but also extends them by providing a dynamic and quantitative framework linking forest sustainability with modern technological interventions.

The study shows that integrating technology and conservation effort can enhance forest biomass growth, supporting and extending existing ecological and technological models. Limitations include the assumption of homogeneous forest conditions, aggregated effects of technology and efforts [29]. Future work could incorporate spatial variability, stochastic events, economic and policy feedbacks, and empirical data from AI-based monitoring to improve accuracy and applicability for sustainable forest management [30].

Conclusions

This study demonstrates that integrating technological interventions with conservation efforts is essential for sustaining and enhancing forestry biomass. The model yields four equilibria, of which only the interior equilibrium is stable; all solution trajectories converge globally to this point. At baseline parameters, the system stabilized at approximately 139 tons/ha of biomass (87% of the carrying capacity), supported by 1.68×10^5 USD in research investment and 6.71×10^8 USD in conservation effort. Parameter experiments revealed clear quantitative effects. Doubling the technology growth rate λ increased biomass by ~0.06 units and raised the technology index by ~167 units. Doubling the carrying capacity coefficient s_2 produced the largest biomass gain (~2.8 units over 50 years). Doubling the effort efficiency

θ raised the equilibrium effort F by ~15-16 units. Sensitivity analysis further confirmed that biomass is most responsive to s_2 and s_1, indicating that capacity expansion and intrinsic growth enhancement exert the strongest influence.

Taken together, these findings show that technology improves the speed and precision of intervention, while ecological measures such as reforestation directly increase biomass potential. A balanced increase in both yields the most sustainable outcomes. The study provides concrete numerical benchmarks for decision-makers: even modest increases in technology-driven conservation (on the order of 10^3-10^6 USD) can yield measurable, long-term improvements in forest biomass. Integrating advanced monitoring technologies with ecological restoration enables more effective forest management, safeguards biodiversity, and supports climate change mitigation.

Additional Information

Author Contributions

All authors have reviewed the final version to be published and agreed to be accountable for all aspects of the work.

Concept and design: Deepika Marwar, Alok Malviya, Shyam Sundar, Maninder Singh Arora

Acquisition, analysis, or interpretation of data: Deepika Marwar, Alok Malviya, Shyam Sundar, Maninder Singh Arora

Drafting of the manuscript: Deepika Marwar

Critical review of the manuscript for important intellectual content: Alok Malviya, Shyam Sundar, Maninder Singh Arora

Supervision: Alok Malviya, Maninder Singh Arora

Disclosures

Human subjects: All authors have confirmed that this study did not involve human participants or tissue. **Animal subjects:** All authors have confirmed that this study did not involve animal subjects or tissue. **Conflicts of interest:** In compliance with the ICMJE uniform disclosure form, all authors declare the following: **Payment/services info:** All authors have declared that no financial support was received from any organization for the submitted work. **Financial relationships:** All authors have declared that they have no financial relationships at present or within the previous three years with any organizations that might have an interest in the submitted work. **Other relationships:** All authors have declared that there are no other relationships or activities that could appear to have influenced the submitted work.

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